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# Anchored expectations, liquidity traps, and secular stagnation[1]

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# Anchored expectations, liquidity traps, and secular stagnation


**Abstract**

Recent liquidity traps in developed economies have been *quiet and persistent*: prolonged stagnation without deflationary spirals or belief-driven fluctuations. This pattern has not been well explained by existing models, which exhibit the nonexistence of steady states under a persistently low natural rate of interest, the global indeterminacy of steady states, the local indeterminacy of the trap steady-state equilibrium, and its instability under learning. This paper revisits a liquidity trap using a New Keynesian framework in which agents' long-run expectations are anchored. We show that if long-run expectations are strongly anchored, (1) a steady state exists for any level of the natural rate; (2) the natural rate selects a steady state: the standard steady state is unique when the natural rate is high, whereas the unintended (trap) steady state, featuring an expected real interest rate above the natural rate and an upward bias in long-run expectations, is unique when the natural rate is low; (3) the trap steady-state equilibrium is locally determinate and stable under learning, so that the resulting stagnation is quiet and persistent. Raising the inflation target, when long-run expectations respond sufficiently to it, can help resolve liquidity traps through a mechanism distinct from those emphasized in existing models.



**JEL classification:** C62; D83; E31; E43; E52

**Keywords:** Liquidity trap; Anchored expectations; Natural rate; Expectational stability; Inflation target

# 1 Introduction

Since the late 1990s, developed economies (in particular, Japan) have experienced prolonged episodes described as liquidity traps, in which policy rates were constrained by the effective lower bound and conventional monetary policy was impaired. Central banks reaffirmed or adopted inflation-targeting policies and deployed unconventional tools to raise inflation expectations. Despite these efforts, the U.S. and the euro area were constrained near the lower bound through much of the 2010s, and Japan remained near the lower bound much longer than the U.S. and the euro area. Summers (2015) describes these phenomena as *secular stagnation*, in which a secular decline in the natural rate of interest rendered the effective lower bound chronically binding in the 2010s. If low growth and low inflation become the new normal in developed economies, liquidity traps will pose a chronic risk of prolonged stagnation (Summers, 2014; Eggertsson et al., 2019).

Against this backdrop, a strand of the literature studies the expectations-driven liquidity trap established by Benhabib et al. (2001b) as a workhorse model for analyzing the prolonged stagnation in developed economies. In particular, as Bullard (2010) argued, Japan had been stuck in a liquidity-trap steady state since the late 1990s, and subsequent studies indicate that this view fits Japan's post-1990s experience (see also Aruoba et al., 2018; Hirose, 2020; Cuba-Borda and Singh, 2024). Using this model, the literature studies policy options: fiscal policies (Mertens and Ravn, 2014; Aruoba et al., 2018), appointing a conservative central banker (Nakata and Schmidt, 2019), and neo-Fisherian policies (Schmitt-Grohé and Uribe, 2017; Bilbiie, 2022; Uribe, 2022).

While these frameworks have been highly influential, they fall short of accounting for the quiet and persistent character of recent stagnation in four respects. First, this class of models has *no steady state* if the natural rate of interest (i.e., the full-employment equilibrium real rate) is sufficiently low (see Eggertsson et al., 2019). Second, even if the natural rate is at a normal level, the models exhibit *global indeterminacy* associated with the coexistence of the two steady states: the *standard*, targeted steady state and the *unintended*, liquidity-trap steady state where the effective lower bound binds. Which steady state prevails is determined by non-fundamental movements in expectations rather than by fundamentals. Third, the unintended steady-state rational expectations equilibrium (REE) is *locally indeterminate* (Benhabib et al., 2001b). Fourth, it is *expectationally unstable* under learning (Evans and Honkapohja, 2005), so that pessimistic beliefs can trigger deflationary spirals away from the steady state. Empirically, however, the United States did not fall into substantial deflation after 2008, despite a large and persistent negative output gap at the lower bound (Hall, 2011). The euro area in the 2010s and, for a much longer period, Japan exhibited similar non-explosive low-inflation dynamics. These models therefore offer little guidance on whether secular stagnation emerges from fundamentals, why it persists without a deflationary spiral, or why its duration differs across economies (see Cochrane, 2024).

This paper revisits liquidity traps in a New Keynesian (NK) framework with two empirical features: a persistently low natural rate of interest and anchored long-run expectations. We assume that agents' long-run expectations (that is, expectations at a distant future horizon) are partially anchored to a reference point, which might represent not only an inflation target, but also other anchors recognized in the literature (e.g., experience, income, education), while the natural rate may fall to very low or negative levels. Within this environment, we study whether the interaction between anchored expectations and low natural rates affects the existence and global indeterminacy of steady states and the local determinacy and learning stability of the steady-state equilibrium.

The anchoring of long-run expectations assumed above is a salient empirical feature of these stagnation episodes, observed alongside the decline in natural rates. One source of this anchoring is the stronger commitment to price stability since the early 1980s and the subsequent adoption of implicit or explicit inflation-targeting frameworks in developed economies (see Bernanke et al., 1999).[1] Other sources come from prior beliefs shaped by agents' individual experiences, income, education, and other background information (see the literature in Section 8). Such anchored expectations explain other features of recent low-inflation environments, including the flattening of the Phillips curve (e.g., Ball and Mazumder, 2011, 2019; Jørgensen and Lansing, 2025), the forward guidance puzzle (Gabaix, 2020), and a marked decline in inflation persistence (Bems et al., 2021). Nevertheless, little is known about the relation between anchored expectations and the quiet and persistent character of secular stagnation.

This paper shows that anchoring of long-run expectations overcomes the four limitations identified above. First, strong anchoring of expectations ensures the existence of a steady state at any level of the natural rate. Second, strong anchoring also eliminates global indeterminacy of steady states and makes steady-state selection depend on the natural rate: under sufficiently anchored long-run expectations, the standard steady state uniquely exists if the natural rate is high enough, whereas the unintended steady state uniquely exists if the natural rate is low. Third, in either case, the equilibrium around the steady state is locally determinate. Fourth, it is stable under learning. In this manner, anchored expectations provide a mechanism through which quiet and persistent stagnation can

[1]Even recently, Powell (2025) notes that long-run inflation expectations in the U.S. appear to remain well anchored close to 2 percent.

arise from fundamentals rather than from self-fulfilling fluctuations. As a policy implication, adjusting the inflation target can help induce a transition from the unintended steady state to the standard one, through a mechanism distinct from those emphasized in existing models.

## 1.1 Related literature

Existing studies have addressed subsets of the limitations of expectations-driven trap models through distinct mechanisms. To the best of our knowledge, this paper is the first to overcome these limitations within a single framework of anchored long-run expectations.

### 1.1.1 Secular stagnation and long-run liquidity traps

This paper is closely related to the literature on long-run liquidity traps. After the global financial crisis, a strand of the literature has explored prolonged stagnation at the zero lower bound. Schmitt-Grohé and Uribe (2017) investigate the *jobless recovery* in the United States by focusing on downward nominal wage rigidity, while Benigno and Fornaro (2018) develop a *stagnation-trap* framework under endogenous growth and nominal wage rigidity. Bianchi and Melosi (2017), in turn, explain the absence of deflation in the United States after the Great Recession through monetary–fiscal regime uncertainty. These studies emphasize non-fundamental movements in expectations, confidence, or policy beliefs. Our paper differs from the literature by focusing instead on a secular decline in the natural rate of interest as a source of prolonged stagnation.

In this sense, this paper is related to the literature on Summers' *secular stagnation* hypothesis. Eggertsson et al. (2019) formalize secular stagnation under a permanently negative natural rate of interest, often rooted in demographics and borrowing constraints, together with downward nominal wage rigidity and a chronically binding zero lower bound. Eggertsson and Egiev (2025) argue that this hypothesis applies to Japan's long stagnation, while Billi et al. (2024) characterize optimal monetary policy when the natural rate is permanently negative. Our contribution to this literature is to show that anchored long-run expectations provide a distinct, fundamentals-based mechanism through which a secular decline in the natural rate can generate quiet and persistent stagnation.

### 1.1.2 Global determinacy and steady-state selection

This paper also contributes to the literature on the global indeterminacy of multiple steady states and steady-state selection. Earlier studies show that the unintended steady state can be eliminated under non-Ricardian fiscal policies (e.g., Benhabib et al., 2001a, 2002; Woodford, 2003; Schmidt, 2016) and under specific monetary policy rules (e.g., Alstadheim and Henderson, 2006; Sugo and Ueda, 2008a). Meanwhile, among the literature on long-run stagnation, Schmitt-Grohé and Uribe (2009) show that the unintended steady state is not eliminated even if the lower bound is removed (see also Schmitt-Grohé and Uribe, 2017; Benigno and Fornaro, 2018). Eggertsson et al. (2019) emphasize that a secular stagnation steady state is not eliminated by raising the inflation target, but by an aggressive fiscal expansion.

The present paper contributes to this literature by showing that sufficiently strong anchoring eliminates the global indeterminacy of multiple steady states and makes steady-state selection depend on the natural rate of interest.

### 1.1.3 Local determinacy and learning stability

Further, this paper is related to the literature on the local determinacy of liquidity-trap equilibrium and its expectational stability under adaptive learning. The indeterminacy of the unintended steady-state equilibrium is emphasized by Benhabib et al. (2001b) (see also Woodford, 2003). The instability of the equilibrium is shown by Evans and Honkapohja (2005), generating deflationary spirals away from the steady state (see also Evans et al., 2008). Adaptive learning has been shown to account for the dynamics of recent data on inflation persistence and inflation expectations (e.g., Malmendier and Nagel, 2016; Carvalho et al., 2023; Bank of Japan, 2016; Maruyama and Suganuma, 2020; Rychalovska et al., 2025, 2026). Eusepi (2010) demonstrates that instability is robust to communication between the central bank and the private sector, while Hommes and Lustenhouwer (2019a,b) show the instability under imperfect credibility of inflation targeting. To fill this gap, Cochrane (2018) shows that the liquidity trap equilibrium becomes determinate if fiscal policy pins down the price level—i.e., under the fiscal theory of the price level. Eusepi (2007) shows that the equilibrium can be determinate and stable under learning if monetary policy rules are forecast-based under a specific utility function. Other studies show a stable liquidity trap under social learning (Arifovic et al., 2018) and under pessimistic expectation formation (Lustenhouwer, 2021). Ascari et al. (2023) show that a stable liquidity-trap equilibrium can arise under simple misspecified forecasting models.

Our contribution to this literature is to show that, under sufficiently strong anchoring, the unintended steady-state equilibrium need not be indeterminate or unstable. When the natural rate is sufficiently low, it can instead be unique, locally determinate, and stable under learning.

### 1.2 Roadmap

The remainder of this paper is organized as follows. The next section presents the baseline New Keynesian model with an effective lower bound and characterizes the benchmark standard and unintended steady states and their existence conditions. Section 3 introduces anchoring of long-run expectations and shows how it changes the existence and properties of the two steady states. Section 4 then examines the local determinacy of associated steady-state equilibria, and Section 5 studies their expectational stability under adaptive learning. Section 6 clarifies the features of the economy under anchored expectations. Section 7 discusses policy implications for escaping the unintended steady state. Section 8 assesses the empirical plausibility of our theoretical results in developed economies. Section 9 illustrates the model's dynamics under anchored long-run expectations and presents counterfactual exercises. The final section concludes.

## 2 Model

This section presents a baseline model, two steady-state REEs, and their existence conditions.

### 2.1 NK model

Our baseline model is the standard New Keynesian model developed in Woodford (2003) and Billi et al. (2024) with an effective lower bound:

$$\begin{aligned} x_t &= -\alpha\left(i_t - \bar{E}_t\pi_{t+1} - r^*\right) + \bar{E}_t x_{t+1} + e_t, && (1)\\ \pi_t &= \kappa x_t + \beta\bar{E}_t\pi_{t+1} + u_t, && (2)\\ i_t &= \max\left\{i^I + \phi_\pi\left(\pi_t - \pi^I\right), i^U\right\}. && (3) \end{aligned}$$

Endogenous variables $x_t$, $\pi_t$, and $i_t$ represent the output gap, inflation rate, and nominal interest rate, respectively. Eq. (1) is a log-linearized IS equation derived from households' optimal choice of consumption. The parameter $r^*$ represents the natural rate of interest. Eq. (2) is the Phillips curve derived from the optimizing behavior of monopolistically competitive firms with Calvo price setting. The exogenous variables $e_t$ and $u_t$ are demand and supply shocks, respectively, and they may be serially correlated, but for simplicity, we assume that they follow independent and identically distributed (i.i.d.) processes. Eq. (3) is the Taylor-type nominal interest rate rule with an effective lower bound $i^U$. Parameter $\pi^I$ is the central bank's inflation target, and we denote $y^I \equiv \left(\left(\frac{1-\beta}{\kappa}\right)\pi^I, \pi^I\right)'$ as an inflation-output target. Parameter $i^I \equiv r^* + \pi^I$ is a nominal interest rate corresponding to the inflation target $\pi^I$. If $i^I + \phi_\pi \left(\pi_t - \pi^I\right) \geq i^U$, the central bank responds to the deviation of $\pi_t$ from $\pi^I$ by following the Taylor principle ($\phi_\pi > 1$); otherwise, the central bank fixes $i_t$ at the lower bound. $\bar{E}_t$ is the operator of agents' conditional expectations at time $t$, which may or may not be rational. In this paper, *long-run expectations* refer to the limit of conditional expectations at an infinite horizon, $\lim_{k\to\infty} \bar{E}_t y_{t+k}$, which conceptually coincides with the unconditional expectations $Ey$. $\alpha > 0$, $\kappa > 0$, and $0 < \beta < 1$ are assumed.

We allow the natural rate of interest $r^*$ to be negative. A broad body of the literature estimates negative natural rates across developed economies (e.g., Eggertsson et al., 2019; Holston et al., 2017; Iiboshi et al., 2022). Among theoretical studies, Eggertsson et al. (2019) demonstrate that the negative natural rate is caused by a declining population. Billi et al. (2024) establish a standard form of the NK model with a negative natural rate of interest, assuming overlapping generations à la Blanchard-Yaari (see Galí, 2021). Following these studies, the present paper allows for the possibility of the negative natural rate in the standard NK model (1)–(3).

For later analysis, the NK model is represented in a matrix form:

$$y_t = A_X + B_X \bar{E}_t y_{t+1} + C_X w_t, \tag{4}$$

where $y_t \equiv (x_t, \pi_t)'$ and $w_t \equiv (e_t, u_t)'$, and the subscript $X \in \{S, U\}$ represents whether the effective lower bound binds ($U$) or not ($S$). If the effective lower bound does not bind ($X = S$), coefficient matrices $\{A_X, B_X, C_X\}$ are given by

$$\begin{aligned} A_S &\equiv \begin{bmatrix} 1 & \alpha\phi_\pi \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} \alpha\pi^I\left(\phi_\pi - 1\right) \\ 0 \end{bmatrix}, \ B_S \equiv \begin{bmatrix} 1 & \alpha\phi_\pi \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \\ C_S &\equiv \begin{bmatrix} 1 & \alpha\phi_\pi \\ -\kappa & 1 \end{bmatrix}^{-1}. \end{aligned}$$

If $X = U$, they are given by

$$\begin{aligned} A_U &\equiv \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} \alpha\left(r^* - i^U\right) \\ 0 \end{bmatrix}, \ B_U \equiv \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \\ C_U &\equiv \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}^{-1}. \end{aligned}$$

## 2.2 Steady-state REEs

Define $\mathbb{E}_t$ the operator of rational expectations. Under rational expectations ($\bar{E}_t = \mathbb{E}_t$), as a benchmark, the model (1)–(3) has two steady-state rational expectations equilibria (REEs) (see Benhabib et al., 2001b). If the lower bound does not bind ($X = S$), there exists a standard REE,

$$y_t = y^S + C_S w_t, \tag{5}$$

where $y^S \equiv \left(x^S, \pi^S\right)'$ is a *standard steady state*. In the absence of anchored expectations,

$$y^S = y^I, \tag{6}$$

and $y^S$ might deviate from $y^I$ in later analysis. If $X = U$, there exists an unintended REE:

$$y_t = y^U + C_U w_t, \tag{7}$$

where $y^U \equiv \left(x^U, \pi^U\right)'$ is an *unintended steady state* (or liquidity trap steady state). In the absence of anchored expectations,

$$y^U = \left(\left(\frac{1-\beta}{\kappa}\right)\left(i^U - r^*\right), i^U - r^*\right)', \tag{8}$$

whereas $y^U$ might deviate from this value in later analysis.

These steady states are illustrated in Figure 1, where solid lines describe the Fisher equation ($i = r^* + \pi^I$) and the policy rule (3). Under rational expectations, long-run expectations $Ey$ are equal to the steady state of each regime.

## 2.3 Limitations

While the literature uses the unintended steady state to describe recent prolonged stagnation, the model has a few features that are inconsistent with observed features of the stagnation. First, both steady states cannot exist if the natural rate of interest $r^*$ is sufficiently low. The standard steady state $y^S$ exists if and only if the effective lower bound does not bind at the steady state, $i^I + \phi_\pi\left(\pi^S - \pi^I\right) \geq i^U$, while the unintended one $y^U$ exists if and only if the lower bound binds at the steady state, $i^I + \phi_\pi\left(\pi^U - \pi^I\right) < i^U$. Under $\phi_\pi > 1$, these conditions are summarized as follows:

**Proposition 1** *In the NK model (1)–(3), the standard and unintended steady states (6) and (8) exist together if and only if*

$$r^* \geq i^U - \pi^I. \tag{9}$$

*Otherwise, there is no steady state.*

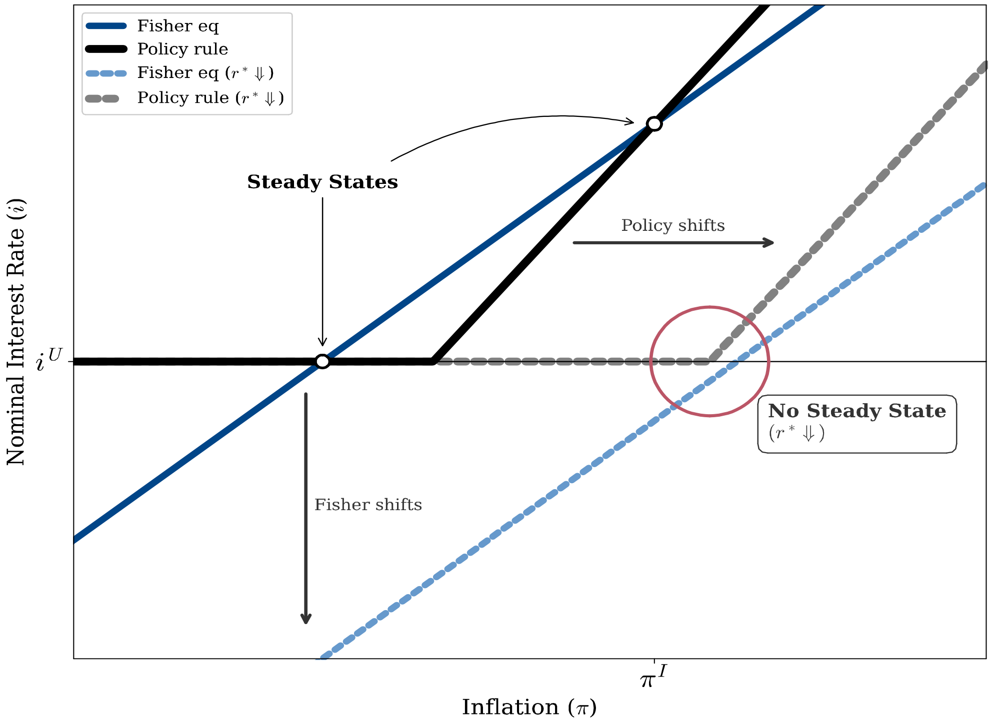


Figure 1: Existence of steady states subject to the natural rate of interest $r^*$: (i) Two steady states under $r^* \geq i^U - \pi^I$ (benchmark, solid lines); (ii) No steady state under $r^* < i^U - \pi^I$ (dotted lines).

This also is illustrated in Figure 1. If the natural rate falls as low as $r^* < i^U - \pi^I$, the Fisher equation shifts to the right and the policy rule shifts downward so that steady states disappear. The nonexistence of steady states under a sufficiently low natural rate is a typical feature of standard NK frameworks (see Eggertsson et al., 2019). However, this feature might not well characterize recent prolonged stagnation because negative natural rates are observed across developed economies (e.g., Eggertsson et al., 2019; Holston et al., 2017; Iiboshi et al., 2022).

Even if Eq. (9) is satisfied, the model exhibits the *global indeterminacy* of multiple steady states so that the unintended steady-state REEs are driven by nothing other than non-fundamental movements in expectations. In addition, the unintended REE is *indeterminate* under rational expectations (Benhabib et al., 2001b) and *expectationally unstable* under learning (Evans and Honkapohja, 2005).

# 3 Anchoring of expectations

This section examines how anchoring of long-run expectations changes the existence and selection of steady states. We show that the degree of anchoring is crucial: when anchoring is sufficiently strong, it fundamentally alters the steady-state properties of the model.

## 3.1 Level anchoring

We introduce *level anchoring* into agents' long-run expectations. The literature finds that long-run expectations may be anchored not only to the central bank's inflation target, but also to other reference values shaped by agents' experience, income, or education. Specifically, we assume that agents' long-run expectations $\bar{E}y$ are partly anchored to a reference value, or *anchor*, $y^A$, while the remainder is determined by rational expectations:

$$\bar{E}y = \lambda_t \, y^A + (1 - \lambda_t) \, \mathbb{E}y. \tag{10}$$

The anchor $y^A$ need not coincide with the inflation-output target $y^I$: it equals $y^I$ when expectations are anchored to the target, but may settle below it ($y^A \leq y^I$) when the central bank persistently fails to achieve the target. For tractability, we assume that the anchor takes the same vector form as the inflation-output target $y^I$: $y^A \equiv \left(\frac{1-\beta}{\kappa}\pi^A, \pi^A\right)'$ where $\pi^A \leq \pi^I$. The parameter $\lambda_t \in [0, 1]$ measures the degree to which long-run expectations are anchored, and both $y^A$ and $\lambda_t$ reflect the full set of anchoring factors rather than the inflation target alone. If $\lambda_t = 0$, expectations reduce to the benchmark of rational expectations, $\bar{E}_t = \mathbb{E}_t$, whereas a larger $\lambda_t$ ties expectations more tightly to the anchor.

This convex-combination form (10) has been used to represent the anchoring of long-run inflation expectations in a range of settings. In particular, Orphanides and Williams (2004) interpret monetary policy as anchoring long-run inflation expectations to a known target within a framework of imperfect knowledge and learning, and the cognitive-discounting model of Gabaix (2020) likewise generates anchoring by shrinking agents' forecasts toward an anchor, namely the steady state.[2] This form is also closely related to empirical anchoring specifications in

[2] The same reduced form arises from distinct microfoundations. In the sticky-information model of Mankiw and Reis (2002), the average expectation is a convex combination of the newly updated forecast and a predetermined one. In the epidemiological model of Carroll (2003),

which inflation forecasts are modeled as a weighted average of a long-run anchor and an informative component (e.g., Ball and Mazumder, 2011, 2019; Hattori and Yetman, 2017; Mehrotra and Yetman, 2018; Carvalho et al., 2023; Hogen and Okuma, 2025; Jørgensen and Lansing, 2025).

We assume that the degree of anchoring $\lambda_t$ has a strictly positive lower bound, $\underline{\lambda} > 0$. For example, the inflation-targeting framework provides a nominal anchor for long-run expectations and, as a form of constrained discretion grounded in transparency and accountability, a credibility foundation that does not rest on realized performance alone (Bernanke et al., 1999). Consistent with this view, Gürkaynak et al. (2010) find that, with the inflation target held fixed, long-run inflation expectations in the United Kingdom became more firmly anchored after the Bank of England was granted operational independence in 1997. The cross-country evidence of Unsal et al. (2022) likewise indicates that central banks with sound frameworks anchor expectations effectively. For Japan, Christensen and Spiegel (2023) find that long-run expectations remained anchored, albeit at a low level below the target, even through a prolonged undershoot and the large shocks of the pandemic (see also Section 8.1). We therefore allow for a positive institutional floor in the strength of anchoring, even when persistent target undershooting substantially erodes the performance-based component of central-bank credibility. Such undershooting can thus endogenously drive $\lambda_t$ toward its institutional floor $\underline{\lambda}$.

Our main analysis conditions on the resulting institutional-floor regime, $\lambda_t = \underline{\lambda}$, rather than on the transition dynamics of $\lambda_t$ toward that regime. This allows us to convey transparently how anchored expectations affect the properties of the trap steady state and its steady-state equilibrium. As shown later, the key characterization of the steady states and equilibria depends on whether the degree of anchoring exceeds a threshold. Provided that the floor $\underline{\lambda}$ exceeds this threshold, the limiting institutional-floor regime remains in the strong-anchoring region. Our analysis below characterizes the equilibrium properties conditional on this regime. This simplification therefore isolates the mechanism of interest, while Appendix A formalizes a simple endogenous credibility mechanism under which $\lambda_t$ can approach the institutional floor underlying the regime analyzed here.

In what follows, we write $\lambda \equiv \underline{\lambda}$ for the constant degree of anchoring in this baseline. Because $e_t$ and $u_t$ are assumed to follow i.i.d. processes, $\bar{E}y$ and $\mathbb{E}y$ coincide with the conditional expectations $\bar{E}_t y_{t+1}$ and $\mathbb{E}_t y_{t+1}$, respectively, so that

households only occasionally absorb professional forecasts, so that the average expectation takes precisely this form. In the regime-switching model of Bianchi and Melosi (2017), long-run expectations are a probability-weighted average of a regime consistent with the announced target and other regimes.

Eq. (10) is transformed to

$$\bar{E}_t y_{t+1} = \lambda\, y^A + (1-\lambda)\, \mathbb{E}_t y_{t+1}. \tag{11}$$

## 3.2 Steady-state equilibria

Substituting the anchored expectations (11) into the NK model (4), we obtain

$$y_t = \left(A_X + \lambda B_X y^A\right) + (1-\lambda)\, B_X \mathbb{E}_t y_{t+1} + C_X w_t.$$

In the non-binding case ($X = S$), a corresponding standard REE takes the form

$$y_t = y^S + C_S w_t, \tag{12}$$

with a standard steady state,

$$y^S = y^A + \frac{\alpha\,(\phi_\pi - 1)\left(\pi^I - \pi^A\right)}{(\phi_\pi - 1)\,\alpha\kappa + \lambda\,(\alpha\kappa + 1 - \beta\,(1-\lambda))} \begin{bmatrix} 1 - \beta\,(1-\lambda) \\ \kappa \end{bmatrix}. \tag{13}$$

In the binding case ($X = U$), a corresponding unintended REE takes the form

$$y_t = y^U + C_U w_t, \tag{14}$$

with an unintended steady state,

$$y^U = y^A + \frac{r^* + \pi^A - i^U}{\lambda - (1-\lambda)\,(\alpha\kappa + \beta\lambda)} \begin{bmatrix} \alpha\,(1 - \beta\,(1-\lambda)) \\ \alpha\kappa \end{bmatrix}. \tag{15}$$

A key implication of Eq. (15) is that the relationship between $y^U$ and $y^A$ is governed by the sign of

$$\lambda - (1-\lambda)\,(\alpha\kappa + \beta\lambda)\,.$$

This sign will be crucial for the existence and selection of steady states below. We therefore define a threshold degree of anchoring, $\bar{\lambda}$, at which this sign changes:

$$\bar{\lambda} \equiv \frac{1}{2\beta}\left(\sqrt{(1-\beta+\alpha\kappa)^2 + 4\alpha\kappa\beta} - (1-\beta+\alpha\kappa)\right).$$

If and only if $\lambda \gtreqless \bar{\lambda}$, then $\lambda - (1-\lambda)\,(\alpha\kappa + \beta\lambda) \gtreqless 0$. When $\lambda = \bar{\lambda}$, the steady-state IS relation (1) becomes parallel to the Phillips curve (2), so that the two steady states cease to exist. In what follows, we abstract from the non-generic case of $\lambda = \bar{\lambda}$ and focus on the two generic cases, $\lambda < \bar{\lambda}$ and $\lambda > \bar{\lambda}$.

### 3.3 Existence

The standard steady state (13) exists if and only if $i^I + \phi_\pi \left(\pi^S - \pi^I\right) \geq i^U$. Define a threshold natural rate $\bar{r}^*$, which changes the sign of $i^I + \phi_\pi \left(\pi^S - \pi^I\right) - i^U$ as follows:

$$\bar{r}^* \equiv i^U - \pi^I + C, \tag{16}$$

where $C \equiv \frac{\lambda\phi_\pi\left(\pi^I - \pi^A\right)(\alpha\kappa + 1 - \beta(1-\lambda))}{(\phi_\pi - 1)\alpha\kappa + \lambda(\alpha\kappa + 1 - \beta(1-\lambda))} \geq 0$. The existence condition is then summarized as follows (see Appendix B for proof):

**Proposition 2** *The standard steady state (13) exists if and only if $r^* \geq \bar{r}^*$, regardless of the degree of anchoring $\lambda$.*

The unintended steady state (15) exists if and only if $i^I + \phi_\pi \left(\pi^U - \pi^I\right) < i^U$, which is expressed as follows (see Appendix C for proof):

**Proposition 3** *The unintended steady state (15) exists if and only if either*

$$\lambda < \bar{\lambda} \textit{ and } r^* \geq \bar{r}^*, \tag{17}$$

*or*

$$\lambda > \bar{\lambda} \textit{ and } r^* < \bar{r}^*. \tag{18}$$

These conditions are illustrated in Figure 2. An important finding is that the existence of both steady states depends jointly on the degree of anchoring $\lambda$ and the natural rate of interest $r^*$. If long-run expectations are anchored as weakly as $\lambda < \bar{\lambda}$, the benchmark pattern in Proposition 1 is preserved: both steady states exist under $r^* \geq \bar{r}^*$ and fail to exist under $r^* < \bar{r}^*$. By contrast, if $\lambda > \bar{\lambda}$, the existence of steady states is fundamentally altered as follows:

**Corollary 1** *Under sufficiently strong anchoring, $\lambda > \bar{\lambda}$, steady-state selection becomes unique and state-dependent: the standard steady state (13) uniquely exists under $r^* \geq \bar{r}^*$, whereas the unintended steady state (15) uniquely exists under $r^* < \bar{r}^*$.*

This result is in stark contrast to the benchmark in three aspects. First, if $\lambda > \bar{\lambda}$, there must exist at least one steady state so that, unlike the benchmark, the economy never exhibits a permanent deflationary spiral. Second, if $\lambda > \bar{\lambda}$, the global indeterminacy of multiple steady states is eliminated. Third, if $\lambda > \bar{\lambda}$, the

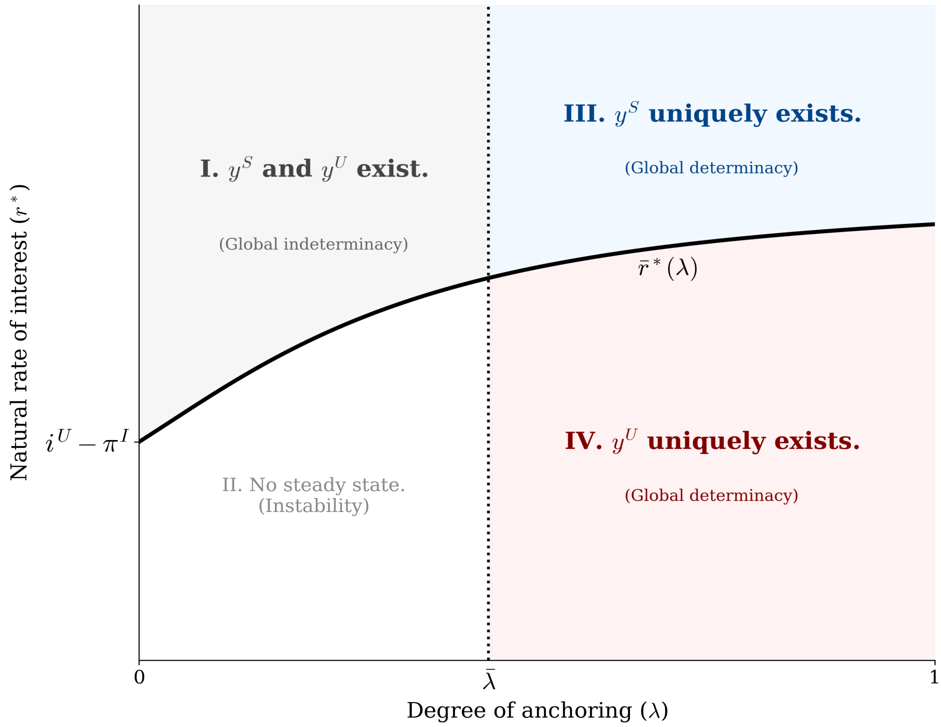


Figure 2: Existence of the standard ($y^S$) and unintended ($y^U$) steady states.

realized steady state is pinned down by the natural rate: if $r^* \geq \bar{r}^*$, the standard steady state (13) obtains; otherwise, the unintended steady state (15) prevails. This state-dependent selection mechanism provides a theoretical foundation for Summers' secular stagnation hypothesis.

### 3.4 Intuition

The intuition behind this mechanism is given by focusing on a steady-state Fisher equation under anchored expectations. At a steady state, $\bar{E}\pi = \lambda\pi^A + (1-\lambda)\,\mathbb{E}\pi$ and $\mathbb{E}\pi = \pi$. Eqs. (1) and (2) provide a (modified) steady-state Fisher equation:

$$i = -\pi\left(\frac{\lambda - (1-\lambda)\,(\alpha\kappa + \beta\lambda)}{\alpha\kappa}\right) + \left(r^* + \pi^A \frac{\lambda\,(1 - \beta\,(1-\lambda) + \alpha\kappa)}{\alpha\kappa}\right). \tag{19}$$

Unlike the standard Fisher equation ($i = r^* + \pi$ when $\lambda = 0$), we find that the modified one (19) has the relationship between $i$ and $\pi$ which depends on the sign of $\lambda - (1-\lambda)(\alpha\kappa + \beta\lambda)$, that is, on whether $\lambda > \bar{\lambda}$ or not:

**Lemma 1** *If $\lambda > \bar{\lambda}$, the Fisher equation (19) is downward sloping with respect to $\pi$; otherwise, the equation is upward sloping, as in the benchmark case.*

The intuition is straightforward. If expectations are anchored as weakly as $\lambda < \bar{\lambda}$, an increase in $i$ passes through almost one-for-one to increases in $\bar{E}\pi$ and $\pi$ at the steady state so that the Fisher equation remains upward sloping. If long-run expectations are anchored as strongly as $\lambda > \bar{\lambda}$, on the other hand, an increase in $i$ raises $i - \bar{E}\pi$ and reduces $\pi$. Hence, strong anchoring makes the Fisher equation downward sloping. This negative relationship is analogous to the negative long-run relationship between the nominal interest rate and inflation that Cochrane (2024) obtains under adaptive expectations or the long-run Phillips curve in a low inflation environment by Eggertsson et al. (2019).

Figure 3 illustrates that the downward slope of the Fisher equation (19) eliminates the global indeterminacy of multiple steady states. Solid lines depict the policy rule (3) and the Fisher equations under weak anchoring ($\lambda < \bar{\lambda}$) and strong anchoring ($\lambda > \bar{\lambda}$). When the Fisher equation is upward sloping, it either intersects the upward-sloping policy rule at two points, generating global indeterminacy, or fails to intersect it altogether. By contrast, once the Fisher equation becomes downward sloping, it intersects the policy rule at a unique point, thereby eliminating global indeterminacy.

Under the downward-sloping Fisher equation, the selection of the steady state is driven by the natural rate of interest $r^*$ (Corollary 1). If $r^*$ is higher than the threshold $\bar{r}^*$, the two equations intersect above the lower bound so that the standard steady state (13) uniquely exists at Point A. The alternative case, where $r^* < \bar{r}^*$, is illustrated by dotted lines. As $r^*$ decreases, both the Fisher equation and the policy rule shift downward so that the intersection also shifts downward. If $r^*$ falls below $\bar{r}^*$, the intersection reaches the lower bound so that the unintended steady state (15) uniquely exists at Point B. In this way, steady-state selection is governed by the natural rate.

In this environment, anchoring long-run expectations to the inflation-output target does not by itself eliminate the risk of a liquidity trap. Corollary 1 continues to hold even when the anchor coincides with the inflation-output target, that is, $y^A = y^I$. Although inflation targets have been adopted in developed economies to anchor long-run expectations, such anchoring does not provide perfect immunity

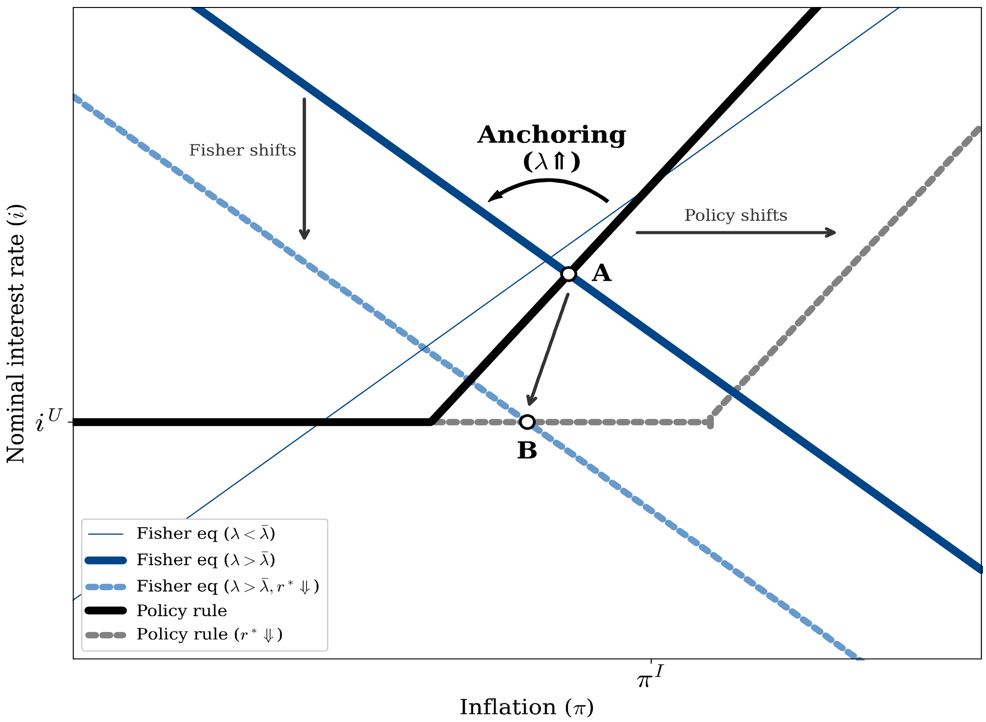


Figure 3: Existence of steady states under anchored expectations: (i) Unique standard steady state under $r^* \geq \bar{r}^*$ (solid lines); (ii) Unique unintended steady state under $r^* < \bar{r}^*$ (dotted lines).

against secular stagnation, and these economies always have a potential to be trapped into prolonged stagnation when natural rates are sufficiently low.

Note that, when the anchor coincides with the target ($y^A = y^I$), the downward slope of the Fisher equation (19) does not overturn the standard Fisher *relation* ($i = r^* + \pi^I$) at the standard steady state. If $y^A = y^I$, the standard steady state features $\pi = \pi^I$, and evaluating (19) at $\pi = \pi^I$ gives $i = r^* + \pi^I$. Thus, even if expectations are strongly anchored, the downward-sloping Fisher equation (19) still passes through the point $\left(\pi^I, r^* + \pi^I\right)$, at which the conventional Fisher relation holds. That is, when $y^A = y^I$, strong anchoring only tilts the Fisher equation (19) to a downward slope.

# 4 Local determinacy

This section examines how anchored long-run expectations affect the local dynamics of the steady-state REEs (12) and (14). In the benchmark, the standard steady-state REE (5) is determinate under rational expectations, whereas the unintended REE (7) is indeterminate so that prolonged adherence to the zero lower bound is expected to generate undesirable, self-fulfilling fluctuations around the unintended steady state (Benhabib et al., 2001b). However, this feature is inconsistent with the empirical evidence of recent prolonged stagnation, where zero interest rate environments persisted without such fluctuations (Cochrane, 2024). We show that the conventional wisdom is modified by anchored expectations.

First, the local dynamics of the standard steady-state REE are not modified (see Appendix E for proof):

**Proposition 4** *If the standard steady-state REE (12) exists, it is determinate under rational expectations, regardless of the degree of anchoring $\lambda$.*

In contrast, the dynamics of the unintended steady-state REE are modified as follows (see Appendix E for proof):

**Proposition 5** *Provided that the unintended steady-state REE (14) exists, it is locally determinate if and only if $\lambda > \bar{\lambda}$.*

This is because anchoring prevents self-fulfilling expectations. If expectations are anchored as weakly as $\lambda < \bar{\lambda}$, expectations can be freely self-fulfilling like the benchmark case without anchoring, leading to the conventional wisdom of local indeterminacy of the unintended REE. As anchoring becomes strong ($\lambda \Uparrow$), anchoring fixes expectations more firmly at an anchor and prevents them from becoming self-fulfilling. If expectations are anchored as strongly as $\lambda > \bar{\lambda}$, expectations are unable to be self-fulfilling so that the unintended REE becomes determinate.

Combined with the existence conditions (Propositions 2 and 3), these results are simplified as follows:

**Corollary 2** *The standard steady-state REE (12) is uniquely determinate if and only if $r^* \geq \bar{r}^*$. The unintended steady-state REE (14) is uniquely determinate if and only if $r^* < \bar{r}^*$ and $\lambda > \bar{\lambda}$.*

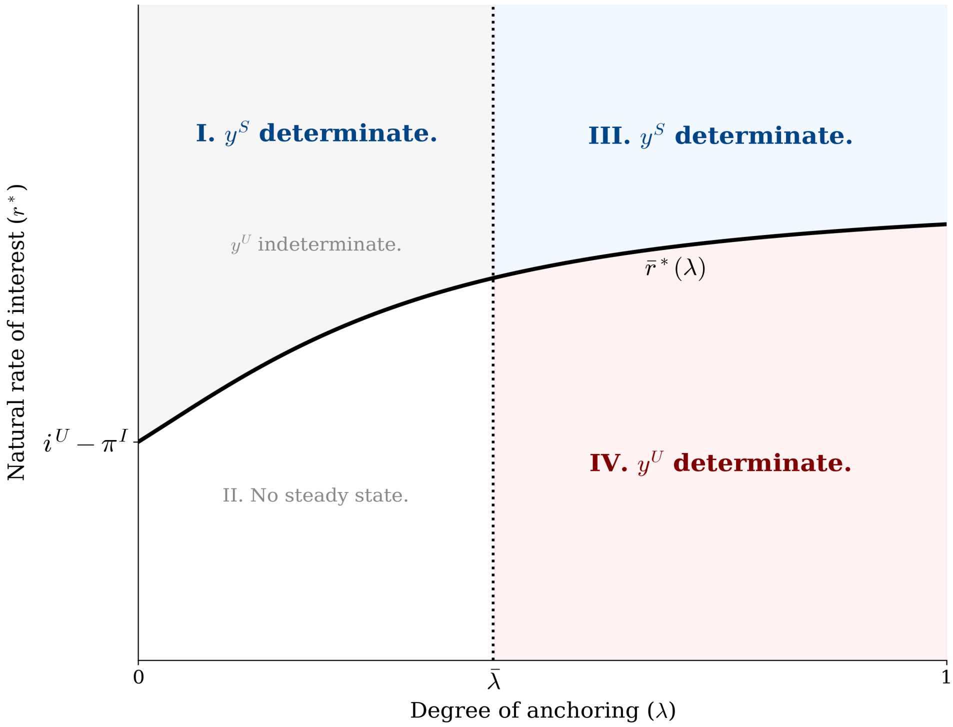


Figure 4: Determinacy of the standard ($y^S$) and unintended ($y^U$) steady-state equilibria.

This is illustrated in Figure 4. The previous section finds that when long-run expectations are strongly anchored, a high natural rate generates a unique standard steady state and a low natural rate eliminates it and generates a unique unintended steady state. We find that the corresponding REE is always determinate around the steady state. This mechanism reinforces the theoretical plausibility of a determinate, fundamentals-driven secular stagnation. Only when the natural rate of interest declines sufficiently, an economy with strongly anchored expectations is prone to falling into a determinate liquidity trap.

Further, Propositions 4 and 5 indicate:

**Corollary 3** *Provided that the standard (or unintended) steady state exists, the* *determinacy/indeterminacy of the corresponding REE (12) (or (14)) is independent of* $y^A$*,* $y^I$*,* $i^U$*, and* $r^*$*.*

That is, the determinacy depends on how strongly long-run expectations are anchored (governed by $\lambda$), but not on where they are anchored (determined by $y^A$).[3] Hence, for example, there can be the case where the unintended steady-state REE is determinate because of the inflation-output target anchoring long-run expectations. Further, our results are robust to whether anchor $y^A$ represents a single value or the combination of multiple values, as long as they serve as fixed reference points for long-run expectations. The empirical literature observes that long-run expectations are anchored to not only, for example, an inflation target, but also other reference points such as experiences and social norms. The proposition suggests that those anchors might have collectively contributed to secular stagnation. Even if an inflation target weakly anchored long-run expectations, other factors might do it firmly enough to generate a secular stagnation. In these cases, parameter $\lambda$ should represent the sum of the degrees of anchoring by multiple different factors.

# 5 Expectational stability

Next, this section examines whether the two steady-state REEs (12) and (14) are stable in a plausible framework of expectations formation rather than rational expectations. Conventional wisdom holds that the standard REE is expectationally stable under learning, whereas the unintended REE is unstable and generates a deflationary spiral away from the steady state (see Evans and Honkapohja, 2005). We show that the unintended REE can instead be stable when long-run expectations are anchored.

## 5.1 Expectations Formation

Following Evans and Honkapohja (2001), we assume that agents form their expectations by partly following econometric learning with the information set of economic variables. Instead of Eq. (11), agents form their expectations as follows:

$$\bar{E}_t y_{t+1} = \lambda y^A + (1-\lambda)\,\tilde{E}_t y_{t+1}, \tag{20}$$

[3] Parameters such as anchor $y^A$ and the natural rate $r^*$ determine the existence of the steady states (as shown in Propositions 2 and 3).

where $\tilde{E}_t$ is the operator of expectations under learning.[4]

Agents form $\tilde{E}_t y_{t+1}$ by estimating a forecasting model of the form of the REE (12) or (14) (that is, the perceived law of motion, PLM),

$$y_t = a + c w_t, \tag{21}$$

where $a \equiv (a_x, a_\pi)'$ is the $2 \times 1$ vector of constant terms and $c$ is the $2 \times 2$ matrix of coefficients for $w_t$. They estimate all parameters and form their forecasts:

$$\tilde{E}_t y_{t+1} = a.$$

Under the assumption that $w_t$ follows i.i.d. processes, $\tilde{E}_t y_{t+1}$ is equal to the constant term $a$, which corresponds to long-run expectations under learning. If exogenous shocks were serially correlated, $\tilde{E}_t y_{t+1}$ could deviate temporarily from $a$ and fluctuate around it. Such extensions, however, do not modify our results.

Under the expectations formation (20), the NK model (1)–(3) provides the actual law of motion (ALM),

$$y_t = T_X(a) + C_X w_t, \tag{22}$$

where

$$T_X(a) \equiv \left(A_X + \lambda B_X y^A\right) + (1-\lambda) B_X a.$$

$T_X(a)$ represents the map from the PLM (21) to the ALM (22) with respect to $a$.

The constant term $a$ is estimated by least squares. Because shocks $w_t$ follow i.i.d. processes, the estimate of parameter $a$ is equal to the sample mean of $y_t$, which is represented by a recursive algorithm (Evans and Honkapohja, 2005):[5]

$$a_t = a_{t-1} + t^{-1}\left(T_X(a_{t-1}) - a_{t-1}\right). \tag{23}$$

The convergence of this algorithm is governed by the associated ordinary differential equation (ODE):

$$\frac{\mathrm{d}a}{\mathrm{d}\tau} = T_X(a) - a, \tag{24}$$

where $\tau$ denotes the notional time. If the ODE is locally asymptotically stable, parameter $a$ converges to the fixed point $\bar{a}$ satisfying

$$T_X(\bar{a}) = \bar{a}.$$

If $a$ converges to $\bar{a}$, parameter $c$ also converges to the fixed point $C_X$ under the assumption that the shocks follow i.i.d. processes (see Evans et al., 2008).

[4]Similar specifications are found in Mehrotra and Yetman (2018), Ball and Mazumder (2019), and Carvalho et al. (2023).

[5]This specification is seen in Ball and Mazumder (2011, 2019), and the literature finds that sample mean forecast rules well describe inflation forecasts of households and professionals (e.g., Coibion and Gorodnichenko, 2012, 2015; Coibion et al., 2018; Bordalo et al., 2020; Jørgensen and Lansing, 2025). Note that stability conditions are robust to constant-gain learning if the gain is sufficiently small (see Evans and Honkapohja, 2001, Theorem 7.9).

## 5.2 Stability

We provide the convergence conditions imposed on parameters. In the non-binding case ($X = S$), the fixed point of the ODE (24) is equal to the standard steady state (13). If the ODE is locally stable, the economy converges to the standard steady-state REE (12), and the REE is said to be expectationally stable under learning (Evans and Honkapohja, 2001).

The ODE (24) is locally stable if and only if its Jacobian,

$$D\left(T_S\left(a\right)-a\right)=\left(1-\lambda\right)B_S-I_2,$$

has all eigenvalues with negative real parts. It provides a parameter condition for the expectational stability of the standard steady-state REE (12) (see Appendix F.1 for proof):

$$\phi_\pi > 1-\frac{\lambda\left(1-\beta\left(1-\lambda\right)+\alpha\kappa\right)}{\alpha\kappa}.$$

We find that the degree of anchoring $\lambda$ relaxes the condition imposed on $\phi_\pi$. This is intuitive because the closer expectations are anchored near the steady state, the more easily they remain around the steady state. Under the assumption of $\phi_\pi > 1$, this condition leads to

**Proposition 6** *The standard steady-state REE (12) is locally stable under learning, regardless of the degree of anchoring $\lambda$.*

In the binding case ($X = U$), the fixed point $\bar{a}$ is equal to the unintended steady state (15). The ODE (24) is locally stable if and only if its Jacobian,

$$D\left(T_U\left(a\right)-a\right)=\left(1-\lambda\right)B_U-I_2$$

has all eigenvalues with negative real parts. It provides a condition for stability (see Appendix F.2 for proof):

**Proposition 7** *The unintended steady-state REE (14) is locally stable under learning if and only if*

$$\lambda > \bar{\lambda}.$$

The stabilizing effect of anchored expectations also works on the unintended steady-state REE (14). If expectations are anchored as weakly as $\lambda < \bar{\lambda}$, the unintended steady state (15) exists under $r^* \geq \bar{r}^*$ but the REE stays unstable under learning. On the other hand, if expectations are anchored as strongly as

$\lambda > \bar{\lambda}$, the unintended steady state exists under $r^* < \bar{r}^*$ and the REE becomes stable. Under $r^* < \bar{r}^*$, the standard steady state does not exist, and the economy necessarily converges to the unintended steady state.

These stability conditions are the same as the determinacy conditions (Propositions 4 and 5). Hence, the local dynamics of the two steady-state REEs are summarized as follows:

**Corollary 4** *The standard steady-state REE (12) is uniquely determinate and expectationally stable under learning if and only if $r^* \geq \bar{r}^*$. The unintended steady-state REE (14) is uniquely determinate and stable if and only if $r^* < \bar{r}^*$ and $\lambda > \bar{\lambda}$.*

This result reinforces that the unintended REE is plausible under anchored long-run expectations in terms of expectational stability as well as local determinacy. When long-run expectations are strongly anchored, a high natural rate generates a unique, determinate, and expectationally stable standard steady-state REE and a low natural rate eliminates it and generates a unique, determinate, and stable unintended steady-state REE. This mechanism is therefore well suited to explaining quiet and persistent stagnation.

Finally, similar to Corollary 3,

**Corollary 5** *Provided that the standard (or unintended) steady state exists, the stability of the corresponding REE (12) (or (14)) is independent of $y^A$, $y^I$, $i^U$, and $r^*$.*

That is, the expectational stability of the REE also comes from the degree of anchoring $\lambda$ rather than the level of anchor $y^A$.

# 6 Features of the steady states

This section shows that the steady states under sufficiently strong anchoring have three plausible properties: (1) anchor-dependent vulnerability to the trap, (2) biases in expectations, and (3) the flattening of the long-run Phillips curve. These properties help clarify how anchored expectations shape the qualitative behavior of stagnation equilibria.

## 6.1 Effect of anchor $y^A$

We first show how the level of the anchor $y^A$ affects steady-state selection under anchored expectations. The threshold of the natural rate $\bar{r}^*$ determines whether the equilibrium converges to the standard steady state or the unintended one. As is shown in Figure 2, a high threshold makes the economy more vulnerable to falling into the unintended steady state and harder to escape from it. Given a degree of anchoring $\lambda$, the threshold $\bar{r}^*$ is affected by the level of the anchor $y^A$ as follows:

**Proposition 8** *Under $y^A \leq y^I$, a lower anchor $\pi^A$ raises the threshold of the natural rate $\bar{r}^*$ required for the economy to remain in the standard steady state. Equivalently,*

$$\frac{d\bar{r}^*}{d\pi^A} \leq 0.$$

That is, the level of the anchor affects steady-state selection through its effect on the threshold $\bar{r}^*$. We refer to this property as *anchor-dependent vulnerability*: a lower anchor $\pi^A$ (or $y^A$) raises $\bar{r}^*$, so that even for a given natural rate $r^*$, an economy with a low anchor is more likely to end up in the unintended steady state. Conversely, a higher anchor lowers $\bar{r}^*$ and thereby reduces Region IV, that is, the range of fundamentals under which the unintended steady state arises. In this sense, the anchor does not merely affect long-run expectations directly; through the equilibrium-selection mechanism, it also governs the economy's anchor-dependent vulnerability to secular stagnation.[6] This mechanism also suggests that differences in anchors can generate cross-country differences in stagnation outcomes even under similar natural-rate environments, a point revisited in Sections 8.2 and 9.5.

## 6.2 Bias in expectations

Under strongly anchored expectations ($\lambda > \bar{\lambda}$), the standard steady state (13) exhibits a downward bias in expectations ($\bar{E}y < y^S$), which is not observed in the absence of anchored expectations.

**Proposition 9** *If $\lambda > \bar{\lambda}$ and $r^* \geq \bar{r}^*$, the standard steady state (13) exhibits $y^I > y^S > \bar{E}y > y^A$ and $i - \bar{E}\pi < r^*$.*

[6]Kuroda (2022), former governor of the Bank of Japan, officially referred to *zero-percent anchoring* of inflation expectations as a key reason why the Bank of Japan failed to achieve its inflation target until the early 2020s.

When the anchor $y^A$ is less than the target $y^I$, $\bar{E}y$ and $y^S$ are necessarily less than $y^I$. This is because the central bank lowers the nominal rate $i$ to lower $i - \bar{E}\pi$ below $r^*$. This gap raises $y^S$ above $y^A$, but $\bar{E}y$ is less than $y^S$ as $\bar{E}y$ is partly anchored to $y^A$. These gaps widen as the degree of anchoring $\lambda$ increases. This is analogous to the *deflationary bias* in Nakata and Schmidt (2019).

The situation is more subtle at the unintended steady state (15). Unlike the standard steady state, the sign of the expectations bias now depends on how low the natural rate is relative to the lower bound $i^U$ and the anchor $\pi^A$.

**Proposition 10** *If $\lambda > \bar{\lambda}$ and $r^* < \bar{r}^*$ (Region IV), the unintended steady state (15) exhibits:*

1. *if and only if $r^* \geq i^U - \pi^A$, then $y^I > y^U \geq \bar{E}y \geq y^A$ and $i - \bar{E}\pi \leq r^*$;*
2. *if and only if $r^* < i^U - \pi^A$, then $y^I > y^A > \bar{E}y > y^U$ and $i - \bar{E}\pi > r^*$.*

If the natural rate is not too low, $r^* \geq i^U - \pi^A$, the unintended steady state exhibits the same downward bias in expectations as the standard steady state in Proposition 9. By contrast, if $r^* < i^U - \pi^A$, the bias reverses: long-run expectations become upward biased relative to realized outcomes, $\bar{E}y > y^U$, and the expected real interest rate exceeds the natural rate, $i - \bar{E}\pi > r^*$. Even more surprisingly, there can be the case where a reduction in $y^A$ causes $y^U$ and $\bar{E}y$ to drop below $y^A$. Its intuition is simple. In the situation where $r^* < i^U - \pi^A$, even if $\bar{E}y$ and $y^U$ are as low as the anchor $y^A$ such that $\bar{E}\pi = \pi^A$, the real interest rate $i - \bar{E}\pi$ is above the natural rate $r^*$. Since the central bank cannot lower the nominal interest rate below the effective lower bound, the high real rate further reduces $y^U$ and $\bar{E}y$ below $y^A$, and $\bar{E}y$ is above $y^U$ by being partly anchored to $y^A$. These gaps widen as the degree of anchoring $\lambda$ increases.

Proposition 10.2 implies that, when $r^* < i^U - \pi^A$, an upward bias in expectations is not necessarily a sign of an impending recovery. Instead, it can be a symptom of a severe stagnation regime in which the effective lower bound prevents the nominal interest rate from falling enough to align the real rate with the natural rate. In this case, upward-biased long-run expectations can coexist with persistently depressed outcomes.

These qualitative implications are broadly consistent with recent stagnation episodes in developed economies. In particular, prolonged stagnation has often been associated with a real rate that remains above the natural rate. At the same time, long-run inflation expectations in developed economies have frequently remained above realized inflation even during low-inflation or deflationary periods (see Carvalho et al., 2023; Weber et al., 2022). Although these patterns have been interpreted through different mechanisms in the literature, the present framework shows that both can arise naturally from anchored expectations.[7]

## 6.3 Flattening of the long-run Phillips curve

A further feature of the steady states under anchored expectations is that strong anchoring flattens the *long-run* Phillips curve, even though the structural slope $\kappa$ of the Phillips curve (2) is left unchanged.

Consider the relationship between steady-state inflation and output gap implied by the anchoring specification (11). At a steady state, the rational component of long-run expectations satisfies $\mathbb{E}_t \pi_{t+1} = \pi$, so that $\bar{E}_t \pi_{t+1} = \lambda \pi^A + (1-\lambda)\pi$. Substituting it into (2) and setting $u_t = 0$ yields the long-run Phillips curve:

$$\pi = \Gamma(\lambda)\, x + \frac{\beta\lambda}{1-\beta(1-\lambda)} \pi^A, \tag{25}$$

where $\Gamma(\lambda) \equiv \frac{\kappa}{1-\beta(1-\lambda)}$ is the slope of the long-run Phillips curve. Anchoring discounts the coefficient on expected inflation in (2) from $\beta$ to $\beta(1-\lambda)$, so that (2) takes the form of a discounted Phillips curve.

**Corollary 6** *A higher degree of anchoring $\lambda$ makes the long-run Phillips curve (25) flatter.*

This is because anchoring mutes the pass-through from current inflation to long-run expectations. When inflation expectations are weakly anchored, current inflation feeds strongly into expected future inflation, amplifying the effect of the output gap on current inflation. By contrast, when expectations are strongly anchored, expected future inflation is less sensitive to current inflation. This weakens the feedback from the output gap to inflation and reduces the long-run slope of the Phillips curve.

This feature is consistent with recent empirical evidence. Hazell et al. (2022) find that the slope of the Phillips curve is small and has changed little since the early 1980s, and attribute the apparent flattening of the aggregate Phillips curve since the 1990s to long-run inflation expectations becoming firmly anchored. Ball

[7] Capistrán and Timmermann (2009) provide a preference-based theory that explains biases in inflation expectations by using asymmetries in preferences. Baqaee (2020) provides a mechanism based on ambiguity aversion. Afrouzi and Veldkamp (2020) provide a belief-based explanation that parameter uncertainty over positively skewed distributions leads to a systematic upward bias in people's beliefs about inflation. Gorodnichenko and Sergeyev (2021) derive an upward bias by assuming an exogenous zero lower bound on inflation expectations.

and Mazumder (2011, 2019) document the associated shift of the U.S. Phillips curve from an accelerationist to a level–level relationship as expectations became anchored at the Federal Reserve's target. Jørgensen and Lansing (2025) provide a closely related New Keynesian analysis in which changes in the degree of anchoring affect the slopes of reduced-form Phillips-curve relationships. Their analysis shows that improved anchoring can account for observed changes in Phillips-curve slopes without necessarily requiring a decline in the structural slope of the New Keynesian Phillips curve. These studies reinforce the plausibility of modeling long-run expectations as partially anchored.

# 7 Escaping the trap: Policy implications

This section explores structural remedies to resolve the determinate and stable unintended steady-state REE under a low natural rate. The previous section raises a fundamental policy question: how can a central bank escape this trap? A possible remedy for escaping a liquidity trap is to lower the threshold $\bar{r}^*$ so that $r^* \geq \bar{r}^*$. We examine how a change in the inflation target affects the threshold $\bar{r}^*$, and thereby the condition under which the economy can escape the unintended steady state.

## 7.1 State-dependent inflation target

To find it, we consider the possibility that the anchor $\pi^A$ responds to the inflation target $\pi^I$. There is much evidence that the anchors of long-run inflation expectations have been correlated with inflation targets in developed economies (see Section 8.2). Let us represent this relationship as $\eta \equiv \frac{d\pi^A}{d\pi^I}$, and define its threshold $\bar{\eta} \equiv \frac{(\phi_\pi - 1)(\lambda - (1-\lambda)(\alpha\kappa + \beta\lambda))}{\lambda\phi_\pi(\alpha\kappa + 1 - \beta(1-\lambda))}$ ($\bar{\eta} > 0$ under $\lambda > \bar{\lambda}$). Using these parameters, the relationship between the threshold of the natural rate $\bar{r}^*$ and the inflation target $\pi^I$ is described as follows (see Appendix G for proof):

**Proposition 11** $\frac{d\bar{r}^*}{d\pi^I} \lesseqqgtr 0$ *if and only if* $\eta \gtreqqless \bar{\eta}$.

That is, the target $\pi^I$ provides a different effect on the threshold $\bar{r}^*$, depending on whether $\eta \geq \bar{\eta}$ or not. If the anchor $\pi^A$ positively responds to the target $\pi^I$ as strongly as $\eta \geq \bar{\eta}$, raising the target lowers the threshold $\bar{r}^*$ and helps the economy escape the unintended steady state.[8] Otherwise, lowering the target may help the economy recover in this conditional case.

This result suggests that adjusting the inflation target can, in principle, facilitate the transition away from the unintended steady state through a shift in steady-state selection. Raising the inflation target has long been advocated by Phelps (1972), Summers (1991), and Fischer (1996), but it aimed to lower the expected real interest rate in response to a *temporary* decline in the natural rate. Krugman (1998) applied this logic to the Japanese deflation in the late 1990s, and Eggertsson and Woodford (2003) formalize it in a New Keynesian framework (see also Schmitt-Grohé and Uribe, 2017). It follows that raising the target has rarely been proposed in the literature on secular stagnation, because this policy does not contribute to recovery from the unintended steady state in those models (e.g., Eggertsson et al., 2019). Our model, by contrast, demonstrates that the central bank may help the economy escape the unintended steady state through a mechanism distinct from those in prior models. Raising the target can eliminate the unintended steady state under strongly anchored expectations and make the standard steady state.

The other finding in Proposition 11 is that lowering the target may be an option if $\eta < \bar{\eta}$. The intuition behind this mechanism is illustrated in Figure 5, which shows the upward-sloping policy rule (3) (black solid line) and the downward-sloping Fisher curve under strongly anchored expectations (19) (blue solid line). In this situation, the steady-state equilibrium is determined at the unintended steady state (Point A). Suppose that the central bank raises the inflation target $\pi^I$ and shifts the policy rule downward (dotted grey line). If the anchor $\pi^A$ responds to the target $\pi^I$ strongly, an increase in $\pi^I$ shifts the Fisher curve significantly upward (light blue dotted line) by lowering $\bar{r}^*$, and the economy recovers to the standard steady state (Point B) without any change in $r^*$. Otherwise, if $\pi^A$ responds weakly, the shift in the Fisher curve is small (light blue dashed line) so that the economy moves to the new unintended steady state (Point C). In this condition, lowering $\pi^I$ contributes to economic recovery.

These results provide a policy implication that the central bank may wish to consider how strongly the inflation target adjusts the anchor of long-run expectations. Much of the literature assumes that the inflation target automatically raises the anchor of long-run expectations. On the other hand, there is much evidence that during Japan's prolonged stagnation, the anchor of inflation expectations was lower than the inflation target (see Section 8). This paper demonstrates that, in

[8] Michau (2025) demonstrates that a depressed economy can escape a permanent liquidity trap through a temporary, but massive, fiscal stimulus to overheat the economy such as to raise an inflation anchor.

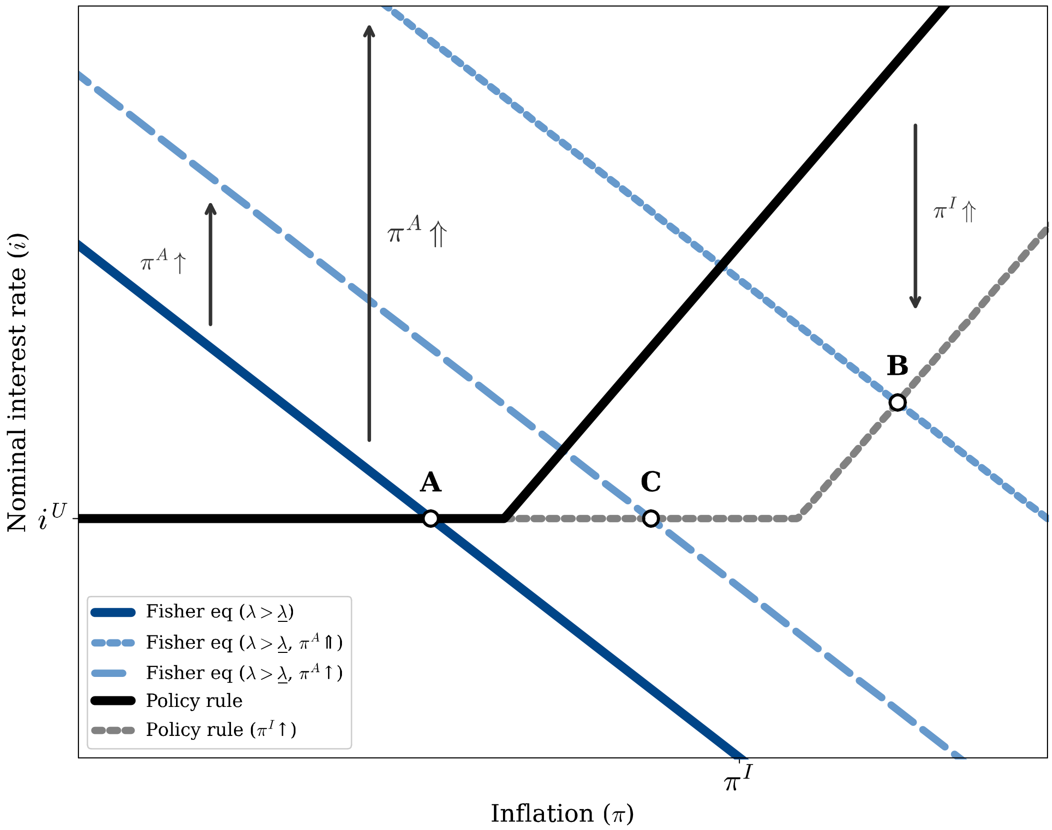


Figure 5: Raising the inflation target $\pi^I$ and the steady state.

such a situation, lowering the target to bring it closer to the anchor of inflation expectations may be an option for the economy to escape the unintended steady state.

## 8 Empirical plausibility

This section examines the empirical plausibility of the model's key mechanism. Specifically, we examine whether strong anchoring ($\lambda > \bar{\lambda}$) and sufficiently low natural rates ($r^* < \bar{r}^*$) are plausible in developed economies before the 2020s. We first calibrate the threshold degrees of anchoring $\bar{\lambda}$'s and compare them with

Table 1: Calibrated thresholds of anchoring $\bar{\lambda}$

| | Region | Sample | $\alpha$ | $\beta$ | $\kappa$ | $\phi_\pi$ | $\bar{\lambda}$ |
|---|---|---|---|---|---|---|---|
| Smets and Wouters (2003) | Euro area | 1980:2-1999:4 | 1/1.39 | 0.99 | 0.011 | 1.69 | 0.08 |
| Smets et al. (2014) | Euro area | 1985:1-2009:4 | 1 | 0.997 | 0.027 | 1.27 | 0.15 |
| Coenen et al. (2025) | Euro area | 1980:1-2019:4 | 1/1.09 | 0.89 | 0.053 | 1.80 | 0.16 |
| Iiboshi et al. (2006) | Japan | 1970:1-1998:4 | 1/2.04 | 0.99 | 0.150 | 1.59 | 0.24 |
| Sugo and Ueda (2008b) | Japan | 1981:1-1995:4 | 1/1.25 | 0.995 | 0.018 | 0.61 | 0.11 |
| Iiboshi et al. (2022) | Japan | 1983:2-2016:2 | 1/1.55 | 0.99875 | 0.051 | 2.07 | 0.17 |
| Smets and Wouters (2007) | U.S. | 1984:1-2004:4 | 1/1.47 | 0.89 | 0.130 | 1.77 | 0.22 |
| Coenen et al. (2025) | U.S. | 1995:1-2019:4 | 1/1.26 | 0.89 | 0.027 | 1.48 | 0.09 |
| Rychalovska et al. (2026) | U.S. | 1981:2-2019:2 | 1/0.92 | 0.998 | 0.011 | 1.28 | 0.10 |

Note: The estimates of structural parameters are provided from Table 1 of Smets and Wouters (2003), Table 2 of Smets et al. (2014), Table 2.1b of Coenen et al. (2025), Table 2 of Iiboshi et al. (2006), Table 2 of Sugo and Ueda (2008b), Table 2 of Iiboshi et al. (2022), "Exit Condition Model" in Table 5 of Smets and Wouters (2007), and "RB" in Table A.2 of Rychalovska et al. (2026). Each $\kappa$ is calculated as $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$ with estimated Calvo parameter $\theta$.

empirical estimates of anchoring strength, $\hat{\lambda}$, in the literature. Next, we calibrate the threshold natural rates $\bar{r}^*$'s and compare them with estimates of $r^*$ in the literature. This section is not a structural estimation of the model. Rather, it provides a plausibility check on whether empirically reasonable parameter values can place the economy in the region emphasized by the theory.

## 8.1 Strong anchoring

First, we examine the plausibility of strong anchoring ($\lambda > \bar{\lambda}$) in developed economies: the euro area, Japan, and the United States. To do so, we calibrate the threshold degrees of anchoring ($\bar{\lambda}$) using structural parameters estimated in the recent literature. Table 1 gathers estimated parameters and calibrated $\bar{\lambda}$'s for these economies. Calibrations indicate $\bar{\lambda} \leq 0.16$ for the euro area, $\bar{\lambda} \leq 0.24$ for Japan, and $\bar{\lambda} \leq 0.22$ for the U.S. At most, about twenty percent of long-run expectations has to be anchored for the plausibility of our mechanism.

Let us compare these country-specific thresholds, $\bar{\lambda}$, with the empirical estimates of anchoring strength, denoted by $\hat{\lambda}$, in the literature. To be precise, $\bar{\lambda}$ should be compared with the institutional floor $\underline{\lambda}$, rather than directly with $\hat{\lambda}$. Because the floor $\lambda$ is not directly observed, this comparison cannot identify whether $\lambda > \bar{\lambda}$. Prolonged target undershooting during the recent trap may nevertheless have pushed realized anchoring strength $\lambda_t$ toward its institutional floor, particu-

Table 2: Estimates of anchoring strength ($\hat{\lambda}$)

| | Anchoring by | |
|---|---|---|
| | Multiple factors | Inflation target |
| Euro area | 0.90 | 0.97 |
| Japan | 0.75 | 0.45 |
| U.S. | 0.95 | 0.77 |

larly in Japan. We therefore use $\hat{\lambda}$ only as a suggestive benchmark for $\lambda$ in this plausibility exercise.

A large body of literature estimates the degrees of anchoring, summarized in Table 2. Among others, Mehrotra and Yetman (2018) estimate that a weight on the level that two-years-ahead inflation forecasts are anchored to is close to 0.90 during the late 1990s to the early 2010s for the euro area, 0.75 for Japan, and 0.95 for the U.S. Estimates for Japan tend to be small (see also Carvalho et al., 2023), and Hogen and Okuma (2025) also estimate a lower weight of 0.58 for Japan (see also Hattori and Yetman, 2017, for Japan during 2000-2015). Although these estimates are based on data of short-run inflation expectations, they all exceed their calibrated thresholds $\bar{\lambda}$ enough to ensure the plausibility of strong anchoring.

One point to note about the above estimates is that they may represent not only the degree of anchoring by central banks, but also the degrees of anchoring by other factors.[9] For these central banks to aim to control long-run expectations, the degree of anchoring by the inflation target alone might be more relevant.[10]

The degrees of anchoring by inflation targets estimated in the literature also are shown in Table 2. For the euro area, Łyziak and Paloviita (2017) estimate the weight of 0.97 on the inflation target in four- or five-year-ahead inflation forecasts during 2007-2015. For Japan, Fukunaga et al. (2025) estimate the weight of 0.45 in ten-year-ahead inflation forecasts. For the U.S., Ball and Mazumder (2019) estimate the weight of 0.77 in inflation forecasts over the next 10 years. We find that even the degrees of anchoring by inflation targets alone may all exceed calibrated $\bar{\lambda}$'s enough to ensure the plausibility of strong anchoring.

[9] See, for example, Madeira and Zafar (2015), Malmendier and Nagel (2016), and Conrad et al. (2022) for U.S. households' lifetime experiences of inflation, Malmendier et al. (2021) for FOMC members' personal inflation histories, Braggion et al. (2024) for German regions with stronger memories of the 1920s hyperinflation, and Diamond et al. (2020) and Fujii et al. (2025) for Japanese households' inflation experiences during the deflationary period since the late 1990s.

[10] See, as empirical evidence of anchoring by inflation targets, Mishkin (2007), Hazell et al. (2022), Binder and Kamdar (2022), and Carvalho et al. (2023) for the United States, Gürkaynak et al. (2010) for the United Kingdom, Kamada et al. (2015), Hattori and Yetman (2017), Fukuda and Soma (2019), and Diamond et al. (2020) for Japan, Beechey et al. (2011), Mehrotra and Yetman (2018), Bottone et al. (2022), and Hoffmann et al. (2022) for euro economies.

Table 3: Natural rates and their thresholds

| | $\pi^A$ | $\bar{r}^*$ | $r^*$ | End of trap |
|---|---|---|---|---|
| Euro area | 1.66 | $-1.41$ | $-2.0$ | 2022 |
| Japan | 1.0 | $-0.05$ | $-4.0$ | 2024 |
| U.S. | 1.93 | $-1.91$ | $-1.5$ | 2015 |

## 8.2 Low natural rates

Next, we examine the plausibility of low natural rates ($r^* < \bar{r}^*$). Before calibrating the threshold natural rate $\bar{r}^*$, we have to determine the perceived long-run inflation anchor ($\pi^A$) for each economy. Table 3 summarizes relevant estimates mentioned below. Mehrotra and Yetman (2018) estimate perceived inflation anchors for two-years-ahead inflation expectations: 1.66% for the euro area and 1.93% for the U.S. over 2010–2014. These estimates may represent anchoring by their inflation targets: The Federal Reserve formalized a 2 percent target in 2012; the ECB adopted a symmetric 2 percent target in 2021 after a long period of "below, but close to, 2 percent." For Japan, on the other hand, Christensen and Spiegel (2023) and Osada and Nakazawa (2024) estimate the long-run anchor of around 1.0% for ten-year-ahead expectations using longer data from 2005 to 2020. Although these estimates deviate from the Bank of Japan's 2% inflation target introduced in 2013, they may represent its implicit targets before 2013.[11] Even after 2013, the 2% target "may not have reflected a substantial change in the BOJ's views on the inflation level consistent with price stability (i.e., 1%)" (see Westelius, 2020, pp. 14–15). In this sense, the above estimates may also partially represent anchoring by the BOJ's target, and Japan's anchor appears substantially lower than those of the euro area and the U.S.

Using these estimates, we calibrate the thresholds $\bar{r}^*$'s for these economies. For expositional clarity, we fix $\left(\pi^I, i^U\right) = (2.0, 0)$ across economies. For the euro area, we obtain $\bar{r}^* = -1.414$ utilizing $\lambda = 0.9$, $\pi^A = 1.66$, and parameters from Coenen et al. (2025) in Table 1. For Japan, we obtain $\bar{r}^* = -0.05$ with $\lambda = 0.75$, $\pi^A = 1.0$, and parameters from Iiboshi et al. (2022). For the U.S., we obtain

[11] In 2000, Bank of Japan (2000) officially defined "price stability" as "a situation which is neither inflationary nor deflationary." (This phrase was used by the Governor Masaru Hayami as early as mid-1998 at a press conference.) In 2006, Bank of Japan (2006) disclosed "an understanding of medium- to long-term price stability," clarifying that its policy board members currently understood the level of price stability from a medium- to long-term viewpoint as being within the range of 0 to 2 percent (with a median of around 1 percent). In 2012, Bank of Japan (2012) introduced "the price stability goal in the medium to long term" of 1% CPI inflation rate (see also Fukunaga et al., 2025).

$\bar{r}^* = -1.91$ with $\lambda = 0.95$, $\pi^A = 1.93$, and parameters from Rychalovska et al. (2026). Despite cross-country differences in structural parameters, calibrations display *anchor-dependent vulnerability* (Proposition 8) such that an economy with a lower anchor $\pi^A$ has a higher threshold $\bar{r}^*$. Since Japan has the highest threshold $\bar{r}^*$ and the U.S. the lowest, Japan seems the most vulnerable to falling into the trap steady state and the hardest to escape from it, whereas the U.S. is the opposite.

Compared with these thresholds, actual natural rates in developed economies might have been sufficiently low over the past decades. In the U.S., the natural rate fell below zero after the global financial crisis until the mid-2010s (e.g., Del Negro et al., 2017), and calibrations place it approximately $-1.5\%$ in that period (e.g., Eggertsson et al., 2019; Benati, 2023). The euro area experienced a similar decline, with estimates of $r^*$ clustering between 0% and $-1\%$ after the crisis (e.g., Holston et al., 2017; Brand et al., 2018) and some measures reaching about $-2\%$ by the late 2010s (e.g., Benati, 2023). Japan's $r^*$ turned negative in the mid-1990s (see also Fueki et al., 2016; Okazaki and Sudo, 2018; Sudo et al., 2018) and plunged to roughly $-4\%$ in the late 2000s (e.g., Iiboshi et al., 2022; Han, 2024). These estimates also are shown in Table 3, suggesting that sufficiently low natural rates ($r^* < \bar{r}^*$) are plausible in the euro area and Japan. The United States is a borderline case: its point estimate ($-1.5\%$) lies slightly above its threshold ($-1.91\%$), the lowest among the three economies.

In summary, strong anchoring ($\lambda > \bar{\lambda}$) is plausible in all three economies, and sufficiently low natural rates ($r^* < \bar{r}^*$) are plausible for the euro area and Japan. Hence, the unintended steady state is plausible for these economies. In particular, Japan with the lowest $r^*$ and the highest $\bar{r}^*$ appears to be the most vulnerable to the trap among the three economies. Section 9.5 illustrates that these differences can generate differences in trap durations across economies.

# 9 Simulations

Using estimated and calibrated parameters, this section simulates the learning dynamics of temporary equilibria under anchored long-run expectations.

## 9.1 Setup

We utilize parameter estimates of $(\alpha, \beta, \kappa, \phi_\pi, \bar{\lambda})$ from Coenen et al. (2025) for the euro area, Iiboshi et al. (2022) for Japan, and Rychalovska et al. (2026) for the U.S. in Table 1. Unless otherwise noted, we assume the policy parameters $(\pi^I, i^U) = (2.0, 0)$. We use the estimates $\hat{\lambda}$ for multiple factors in Table 2 as calibrated

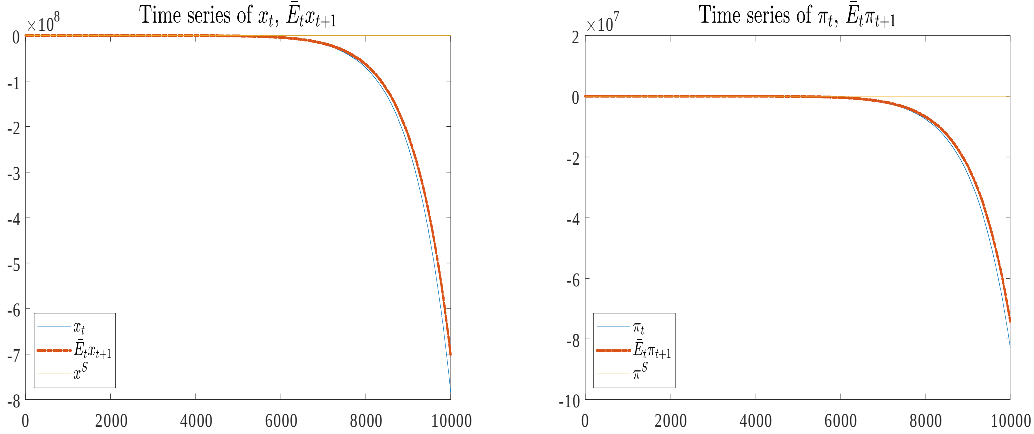


Figure 6: Simulations for the temporary equilibrium and expectations under learning in Region I ($\lambda < \bar{\lambda}$ and $r^* \geq \bar{r}^*$).

values of the model parameter $\lambda$, and take $\left(\pi^A, \bar{r}^*, r^*\right)$ from Table 3. We calculate the updating of the constant term parameters $(a_x, a_\pi)$ in recursive least-squares estimations (23) and the corresponding paths for the temporary equilibrium $(x_t, \pi_t)$ and long-run expectations $\left(\bar{E}_t x_{t+1}, \bar{E}_t \pi_{t+1}\right)$. We use a decreasing-gain algorithm in the simulations.[12] For simplicity, we assume only the demand shock $e_t$ to evolve and follow $N\,(0, 0.1)$.

## 9.2 Weak anchoring

First, we examine how the degree of anchoring $(\lambda)$ affects the dynamics of the temporary equilibrium. As a benchmark, we consider the case of no anchored expectations $(\lambda = 0)$. Using the estimated U.S. parameters, we set $r^* = 2.0$ as a normal condition, placing the economy in Region I of Figure 2. In this region, the standard steady-state REE is stable whereas the unintended REE is unstable, generating global indeterminacy: $y^S = (0.364, 2.0)'$ and $y^U = (-0.364, -2.0)'$.

Figure 6 shows simulations of the temporary equilibrium $y_t$ and expectations $\bar{E}_t y_{t+1}$. The initial values of the constant terms $(a_{x,0}, a_{\pi,0})$ are set at the unintended steady state $y^U$. The simulations reproduce the conventional result that the expectations-driven liquidity trap is unstable (see Evans and Honkapohja,

[12] Following Evans and McGough (2020), we use a decreasing gain sequence with gain $\gamma_t = t^{-\delta}$ $(0 < \delta < 1)$ rather than $\gamma_t = t^{-1}$ to increase the speed of convergence.

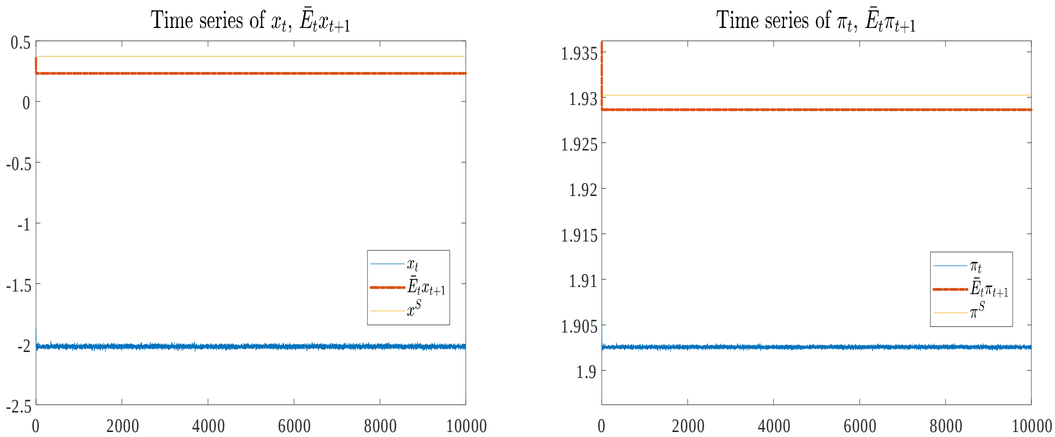


Figure 7: Convergence to the unintended steady state in Region IV ($r^* < \bar{r}^*$ and $\lambda > \bar{\lambda}$).

2005). If initial shocks happen to push expectations downward so that the effective lower bound binds, the temporary equilibrium does not converge to $y^U$, but diverges downward without bound, generating a *deflationary spiral*.[13] The equilibrium converges to the standard steady state $y^S$ only if initial shocks happen to push expectations upward. Overall, when long-run expectations are weakly anchored, the economy does not remain persistently around the unintended steady state.

## 9.3 Strong anchoring

If long-run expectations are strongly anchored ($\lambda > \bar{\lambda}$), the dynamics of the temporary equilibrium are modified. When $r^* = 2.0$ such that $r^* \geq \bar{r}^*$, the economy is in Region III, where there exists the unique standard steady state $y^S = (0.373, 1.93)'$ and the steady-state equilibrium is locally determinate and stable under learning. For any initial values $(a_{x,0}, a_{\pi,0})$, the temporary equilibrium immediately converges to the standard steady state.

This situation is overturned if the natural rate $r^*$ declines as low as $r^* < \bar{r}^*$. If, for example, $r^* = -4.0$, the economy is in Region IV, where there exists the unique unintended steady state $y^U = (-2.019, 1.903)'$ and the steady-state equilibrium is

[13]Note that if we assume $r^* = -3.0$ such that there exists no steady state (Region II), the equilibrium always exhibits a deflationary spiral.

locally determinate and stable. We set the initial values $(a_{x,0}, a_{\pi,0}) = (100, 100)$ to allow an equilibrium to diverge somewhere other than the unintended steady state. Figure 7 shows that the temporary equilibrium immediately converges downward to the unintended steady state and stays around it as long as the natural rate is sufficiently low.

In addition, we find the features of the unintended steady state under the low natural rate. At the steady state, we observe an upward bias in expectations ($\bar{E}y > y$), which is a symptom of severe stagnation instead of a sign of future recovery. In addition, $\bar{E}\pi < 2.0$ so that the expected real interest rate is greater than the natural rate ($i^U - \bar{E}\pi > r^*$). In response, the steady-state output gap $x^U = -2.019$ is significantly lower than the output-gap target $x^I = \frac{1-\beta}{\kappa}\pi^I = 0.364$, although the steady-state inflation rate $\pi^U = 1.903$ is close to the inflation target $\pi^I = 2.0$. This reflects the flattening of the long-run Phillips curve. A liquidity trap can occur even if long-run expectations are strongly anchored to the inflation target.

## 9.4 Secular declines in $r^*$

Next, we simulate how the economy responds to an exogenous evolution of the natural rate under strongly anchored expectations ($\lambda > \bar{\lambda}$). Using the U.S. parameters in previous section, we assume the natural rate to evolve step by step from 2% to $-2$% at $t = 2000$, to $-1.5$% at $t = 4000$, to $-0.5$% at $t = 6000$, and to 2% at $t = 8000$. Given $\bar{r}^* = -1.91$, the unintended steady state uniquely exists in $t \in [2000, 3999]$ and the standard steady state uniquely exists in other periods.

Figure 8 simulates the temporary equilibrium in response to the evolution in the natural rate. We find that under strongly anchored expectations, steady-state selection is governed by fundamentals rather than self-fulfilling expectations. At $t = 2000$, the natural rate declines from 2% to $-2$% enough to ensure $r^* < \bar{r}^*$, the economy falls into secular stagnation by diverging from the standard steady state that disappears and converging to the unintended one that appears. During the stagnation, an upward bias in expectations and a high real interest rate appear. However, the economy stays in a liquidity trap only when the natural rate is sufficiently low. When the natural rate recovers at $t = 4000$ from $-2$% to $-1.5$% enough to ensure $r^* \geq \bar{r}^*$, the equilibrium also recovers from the unintended steady state that disappears and converges to the standard one that appears. Afterward, although the natural rate further recovers from $-1.5$% to 2%, the equilibrium shows no response as the central bank controls the nominal interest rate in response to the natural rate.

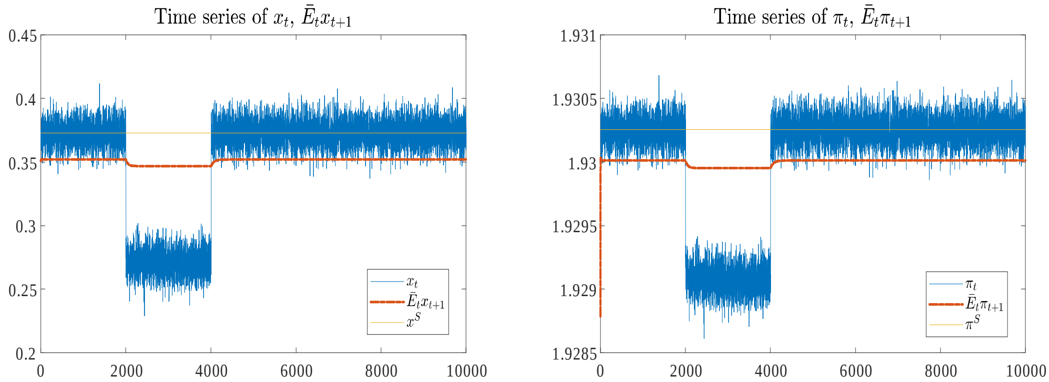


Figure 8: Simulation for a temporary equilibrium in response to an evolution of the natural rate of interest $r^*$ under $\lambda > \bar{\lambda}$, using the U.S. parameters.

## 9.5 Duration of secular stagnation

This section further illustrates how the duration of secular stagnation depends on the level of the anchor $\pi^A$ through its effect on the threshold $\bar{r}^*$. Section 8.2 observes *anchor-dependent vulnerability*: since Japan had the lowest anchor $\pi^A$ (and hence the highest threshold $\bar{r}^*$) and the U.S. the highest, Japan seems the most vulnerable to falling into the trap steady state and the hardest to escape from it, whereas the U.S. is the opposite. Using the parameters in Section 8.2, this section conducts simulations for the euro area and Japan similar to the U.S. simulations in the previous section. We assume the same natural rate path for the three economies.

Figure 9 shows simulations for the euro area (Panel (a)) and Japan (Panel (b)). Similar to the U.S. economy (Figure 8), Panel (a) shows that the equilibrium for the euro area falls from the standard steady state to the unintended one at $t = 2000$ when the natural rate declines from 2% to $-2\%$ enough to ensure $r^* < \bar{r}^*$. During the stagnation, an upward bias in expectations and a high real interest rate appear. However, the durations of secular stagnation are different between the two economies. While the U.S. economy fully recovers to the standard steady state at $t = 4000$ when the natural rate recovers from $-2\%$ to $-1.5\%$, the euro area stays around the unintended steady state as $r^* < \bar{r}^*$ still holds. The equilibrium fully recovers at $t = 6000$ when the natural rate recovers from $-1.5\%$ to $-0.5\%$ enough to ensure $r^* \geq \bar{r}^*$.

Panel (b) simulates a longer duration of Japan's secular stagnation. Similar to

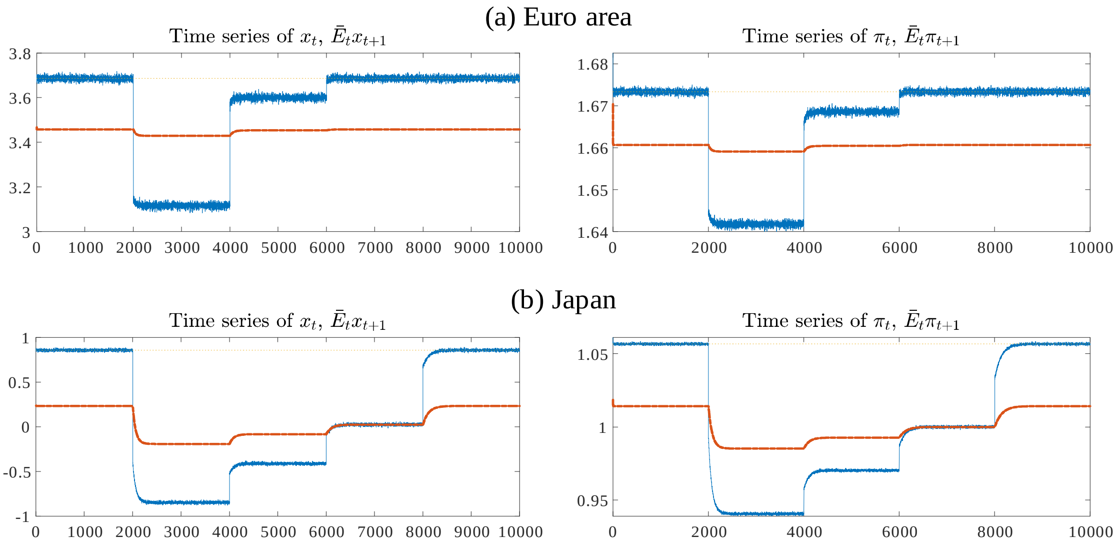


Figure 9: Simulations for the temporary equilibrium in response to an evolution of the natural rate of interest $r^*$ under $\lambda > \bar{\lambda}$, using the parameters for the euro area and Japan.

the other economies, the equilibrium falls from the standard steady state to the unintended one at $t = 2000$ and shows an upward bias in expectations. However, the equilibrium does not recover until $t = 8000$ when the natural rate recovers to 2% enough to ensure $r^* \geq \bar{r}^*$.

These implications are consistent with the observed timing of exits from liquidity traps. Dating each episode by the period during which the policy rate was held at or near its effective lower bound, the United States was in a liquidity trap from 2008 to 2015 (about 7 years), the euro area from 2008 to 2022 (about 14 years), and Japan from 1995 to 2024 (about 29 years). The exit years follow the ordering implied by the estimated anchors $\pi^A$ and the resulting thresholds $\bar{r}^*$ (Table 3): the United States, with the highest anchor $\pi^A$, exited first; the euro area next; and Japan, with the lowest $\pi^A$, last. Consistent with this, Benigno et al. (2024) find that the natural rates of interest in the United States and the euro area have recovered since the early 2020s, while Nakano et al. (2024) provide evidence that Japan's natural rate remains negative in the 2020s and is below the rates in the other two economies. Our simulations thus suggest that the differences in the durations of secular stagnation may stem not only from differences in the paths of natural rates but also from differences in the anchor $\pi^A$.

Further, these results imply that an economy faces a high risk of a persistent liquidity trap when the central bank sets a *low* inflation target under a given natural rate. Note that a lower target can reduce the anchor $\pi^A$ and hence raise the threshold $\bar{r}^*$ and the risk of a liquidity trap. Modern central banks in developed economies set 2% inflation targets uniformly, irrespective of the levels of their own natural rates. To avoid a persistent liquidity trap, however, the appropriate inflation target may differ across countries, depending on the level of the natural rate. This possibility is illustrated in the next subsection.

## 9.6 Counterfactual policy experiments

Finally, this section illustrates the effect of the state-dependent inflation target proposed in Section 7. We perform counterfactual simulations to clarify how adjusting the inflation target affects the duration of secular stagnation.

To do it, we add an event of changing the inflation target $\pi^I$ to the previous simulations for Japan, where the economy fell into the unintended steady state over the longest period ($t = 2000 \sim 8000$) (Panel (b) of Figure 9). Suppose that the central bank raises the target $\pi^I$ from 2.0% to 4.0% at $t = 5000$, and that an increase in the target $\pi^I$ raises the anchor $\pi^A$ one-for-one ($\eta = 1$). This means that $\pi^A$ responds to $\pi^I$ as strongly as $\eta \geq \bar{\eta}$. In this condition, this policy raises $\pi^A$ from 1.0% to 3.0%, which reduces $\bar{r}^*$ from $-0.05$ to $-2.05$ so that the unintended steady state disappears and the standard steady state appears at $t = 5000$ when $r^* = -1.5$.

Figure 10 illustrates this simulation. The economy plunges into the unintended steady state after the decline in the natural rate at $t = 2000$. In contrast to Panel (b) of Figure 9 (no change in the target), the equilibrium immediately jumps up at $t = 5000$ from an unintended steady state to a new standard steady state. At the same time, an upward bias in expectations is transformed into a downward bias. In this way, raising the target contributes to the recovery when the anchor $\pi^A$ strongly responds to the inflation target $\pi^I$.

On the other hand, the result is reversed when $\eta < \bar{\eta}$. Suppose that a change in the target $\pi^I$ does not move the anchor $\pi^A$ ($\eta = 0$). This means that $\pi^A$ responds to $\pi^I$ as weakly as $\eta < \bar{\eta}$. We assume that the central bank lowers the target $\pi^I$ from 2.0% to 1.5% at $t = 5000$.[14] This policy does not affect the anchor $\pi^A$, but

[14] Note that our model assumes that the inflation target $\pi^I$ is not less than the anchor $\pi^A$,

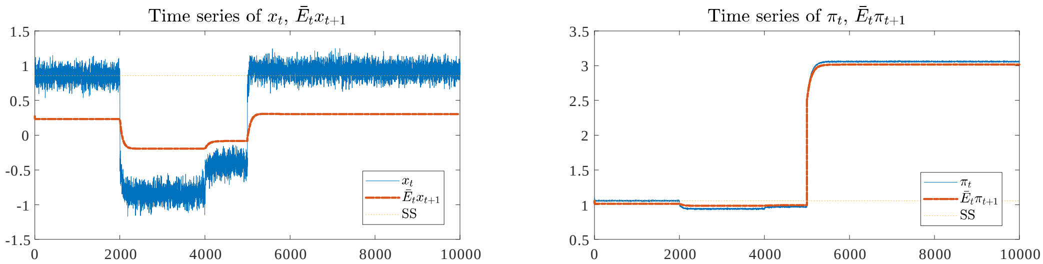


Figure 10: Simulation for raising the inflation target $\pi^I$ from 2.0% to 4.0% at $t = 5000$ with $\eta \geq \bar{\eta}$ and $\lambda > \bar{\lambda}$.

lowers $\bar{r}^*$ from $-0.05$ to $-0.52$ so that the unintended steady state disappears and the standard steady state appears at $t = 6000$ when the natural rate increases from $-1.5\%$ to $-0.5\%$.

Figure 11 illustrates this simulation. Lowering the target at $t = 5000$ raises the economy from the unintended steady state to a new standard steady state at $t = 6000$ when the natural rate increases from $-1.5\%$ to $-0.5\%$.

These results suggest that policy implications depend on how strongly the inflation target affects the anchor of long-run expectations. If the anchor responds strongly to the target, a state-dependent inflation target may help align steady-state selection with the natural rate. If the response is weak, lowering the target may, in some cases, reduce the risk of a persistent liquidity trap. As a rough experiment, we take the estimated weight of long-run expectations on the inflation target alone (Table 2) as a proxy for the parameter $\eta$, under the assumption that this loading approximates the marginal response of the anchor to a change in the target. Using parameter values for the three economies, we calibrate $\bar{\eta} = 0.42$ for the euro area, $\bar{\eta} = 0.49$ for Japan, and $\bar{\eta} = 0.22$ for the U.S. The condition $\eta \geq \bar{\eta}$ holds for the euro area ($0.97 > 0.42$) and the U.S. ($0.77 > 0.22$), whereas it fails for Japan ($0.45 < 0.49$). Hence, raising the inflation target is likely to lower the thresholds in the U.S. and the euro area, while such a policy might backfire in Japan, consistent with the observation that Japan stayed in the trap even after the adoption of the 2 percent target in 2013.

which is 1.0% for Japan.

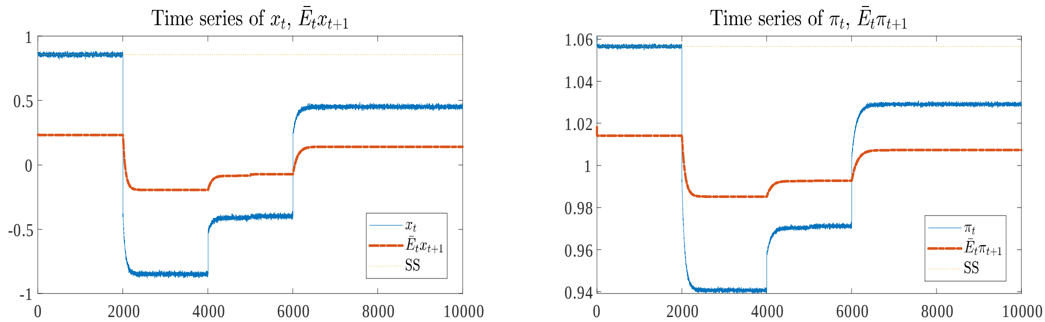


Figure 11: Simulation for reducing the inflation target $\pi^I$ from 2.0% to 1.5% at $t = 5000$ with $\eta < \bar{\eta}$ and $\lambda > \bar{\lambda}$.

# 10 Conclusion

This paper has re-examined the existence, determinacy, and expectational stability of liquidity traps within a New Keynesian framework where long-run expectations are anchored. Our analysis reveals a stark result: when expectations are strongly anchored, a steady state exists for any level of the natural rate, and the global indeterminacy typically associated with multiple steady states is eliminated. In this environment, steady-state selection is governed by fundamentals rather than self-fulfilling sunspots: the standard steady state is unique when the natural rate of interest is high, whereas the unintended steady state is unique when the natural rate is low. In either case, the steady-state equilibrium is locally determinate and stable under learning. In short, strong anchoring changes the nature of the liquidity trap itself: from a volatile, fragile, and belief-driven one into a quiet, persistent, and fundamentals-driven one.

The unintended steady state provides a theoretical foundation for *quiet and persistent* stagnation and is consistent with empirical patterns such as an expected real interest rate above the natural rate and a persistent upward bias in long-run expectations. Numerical simulations illustrate how a secular decline in the natural rate can generate a prolonged liquidity trap, consistent with the experiences of Japan and other developed economies. Policy experiments further show that adjusting the inflation target can shift steady-state selection and, under appropriate conditions, eliminate the unintended steady state. Whether raising or lowering the target facilitates a transition to the standard steady state depends jointly on the natural rate and on how strongly the long-run anchor responds to the target. Thus, adjusting the inflation target offers central banks a potential tool for addressing secular stagnation through expectations management, but both its effectiveness and the appropriate direction of adjustment are contingent on these conditions.

Future research should further endogenize the anchoring process by jointly modeling the dynamics of the anchoring strength $\lambda_t$ and the anchor $y^A$, as well as their responses to policy changes.

# Appendix

# A Endogenous credibility and the institutional-floor regime

We provide a simple endogenous-credibility specification for the anchoring strength $\lambda_t$ that can rationalize the institutional-floor regime analyzed in the main text. To make this mechanism concrete, we consider the special case in which the anchor is the inflation-output target, $y^A = y^I$, and allow $\lambda_t$ to vary with the central bank's performance-based credibility. For simplicity, we give the updating mechanism a representative-agent interpretation: $\lambda_t$ denotes the agent's time-varying credibility weight, while $\underline{\lambda}$ is its institutional floor.

Write the degree of anchoring as

$$\lambda_t = \underline{\lambda} + (1 - \underline{\lambda})\Lambda_t, \qquad \Lambda_t \in [0, 1], \quad \underline{\lambda} \in (0, 1),$$

where $\underline{\lambda}$ denotes an institutionally sustained lower bound on credibility, reflecting central-bank independence, the legal mandate, and the target framework (Bernanke et al., 1999; Gürkaynak et al., 2010; Unsal et al., 2022), and $\Lambda_t$ is a track-record-based component. The average expectation, Eq. (11), holds with this $\lambda_t$.

Following the endogenous-credibility mechanism of Hommes and Lustenhouwer (2019a), and taking the non-anchored predictor to be rational expectations, the representative agent's credibility evolves as

$$\lambda_t = \underline{\lambda} + (1 - \underline{\lambda})\frac{1}{1 + \exp(-\beta_c \Delta U_{t-1})}, \qquad \Delta U_{t-1} = U^A_{t-1} - U^R_{t-1}, \tag{A.1}$$

where $U^A$ and $U^R$ measure the forecasting performance of the target-based and rational predictors for inflation, respectively, and $\beta_c$ is the intensity of choice. When persistent target misses cause the target-based predictor to underperform the rational predictor, $\Delta U_{t-1}$ falls and Eq. (A.1) lowers $\lambda_t$ toward its institutional floor.

Persistent target failure can drive $\lambda_t$ toward its institutional floor $\underline{\lambda}$, providing an endogenous foundation for the low-credibility regime considered in the main text. Hence, conditioning on $\lambda_t = \underline{\lambda}$ can be viewed as a reduced-form representation of the low-credibility limit of this endogenous process.

Because $\underline{\lambda} > 0$, credibility does not collapse to the rational-expectations benchmark $\lambda_t = 0$. If, moreover, $\underline{\lambda} > \bar{\lambda}$, the economy remains in the strong-anchoring regime even when credibility falls to its institutional floor. Conditioning on this regime, Corollary 1 and the results of Sections 4 and 5 imply that the standard steady-state REE is unique, determinate, and stable under learning when $r^* \geq \bar{r}^*$, whereas the unintended steady-state REE has these properties when $r^* < \bar{r}^*$. The estimates reported in Section 8.1 suggest that Japan operated in the strong-anchoring region during the sample period, although they do not directly identify the institutional floor $\underline{\lambda}$.

# B Proof of Proposition 2

The standard steady state (13) exists if and only if the policy rule (3) evaluated at that steady state is not constrained by the effective lower bound:

$$i^I + \phi_\pi \left(\pi^S - \pi^I\right) \geq i^U. \tag{B.1}$$

Eq. (13) provides

$$\pi^S - \pi^I = -\frac{\lambda \left(\pi^I - \pi^A\right) \left(\alpha\kappa + 1 - \beta\left(1-\lambda\right)\right)}{\left(\phi_\pi - 1\right)\alpha\kappa + \lambda\left(\alpha\kappa + 1 - \beta\left(1-\lambda\right)\right)}.$$

Hence,

$$\begin{aligned} i^I + \phi_\pi(\pi^S - \pi^I) &= \pi^I + r^* - \frac{\lambda\phi_\pi \left(\pi^I - \pi^A\right) \left(\alpha\kappa + 1 - \beta\left(1-\lambda\right)\right)}{\left(\phi_\pi - 1\right)\alpha\kappa + \lambda\left(\alpha\kappa + 1 - \beta\left(1-\lambda\right)\right)} \\ &= \pi^I + r^* - C \\ &= r^* - \bar{r}^* + i^U. \end{aligned}$$

Therefore, Eq. (B.1) is transformed to $r^* \geq \bar{r}^*$.

# C Proof of Proposition 3

The unintended steady state (15) exists if and only if the policy rule (3) at the steady state lies below the effective lower bound:

$$i^I + \phi_\pi(\pi^U - \pi^I) < i^U. \tag{C.1}$$

Eq. (15) provides

$$\pi^U - \pi^I = -\left(\pi^I - \pi^A\right) + \frac{\alpha\kappa\left(r^* + \pi^A - i^U\right)}{\lambda - (1-\lambda)(\alpha\kappa + \beta\lambda)}.$$

Hence, Eq. (C.1) is transformed to

$$\frac{r^* - \bar{r}^*}{\lambda - (1-\lambda)(\alpha\kappa + \beta\lambda)} < 0,$$

which therefore provides Eqs. (17) and (18).

# D Obtaining features of the unintended steady state

If and only if $\lambda > \bar{\lambda}$ and $r^* < \bar{r}^*$, the determinate unintended steady state (15) exists and Eq. (11) leads to $\bar{E}y_{t+1} - y = \lambda\left(y^A - y\right)$. Hence,

$$\bar{E}y_{t+1} \gtreqless y \Longleftrightarrow y^A \gtreqless y \Longleftrightarrow r^* + \pi^A \lesseqgtr i^U. \tag{D.1}$$

Therefore, if and only if $i^U - \pi^A \leq r^* < \bar{r}^*$, then $y \geq \bar{E}y_{t+1}$. If and only if $r^* < i^U - \pi^A$, then $y < \bar{E}y_{t+1}$.

Next, Eq. (D.1) provides $\bar{E}x_{t+1} > x_t$ if and only if $r^* < i^U - \pi^A$, and the Euler equation (1) (at the steady state) leads to

$$\begin{aligned} \alpha\left(i - \bar{E}\pi_{t+1} - r^*\right) &= \bar{E}x_{t+1} - x \\ i - \bar{E}\pi_{t+1} &> r^*. \end{aligned}$$

Therefore, the proposition is proved.

# E Determinacy conditions

Following Woodford (2003, pp. 670–671), the determinacy conditions of the two steady-state REEs are provided as follows.

## E.1 Standard steady-state REE

Define $J_I \equiv \left((1-\lambda) B_I\right)^{-1}$. The standard steady-state REE (12) is determinate if and only if matrix $J_I$ has both eigenvalues outside the unit circle, that is, either

$$\text{Case 1: } \begin{cases} \det(J_I) > 1 \Longleftrightarrow \phi_\pi > -\frac{1}{\alpha\kappa}\left(1 - \beta(1-\lambda)^2\right), \\ \det(J_I) - \mathrm{tr}(J_I) > -1 \Longleftrightarrow \phi_\pi > 1 - \lambda - \frac{\lambda(1-\beta(1-\lambda))}{\alpha\kappa}, \\ \det(J_I) + \mathrm{tr}(J_I) > -1 \Longleftrightarrow \phi_\pi > -\frac{1}{\alpha\kappa}\left(1 + (1-\lambda)(1 - \beta\lambda + 2\beta + \alpha\kappa)\right), \end{cases}$$

or

$$\text{Case 2: } \begin{cases} \det(J_I) - \operatorname{tr}(J_I) < -1 \Longleftrightarrow \phi_\pi < 1 - \lambda - \frac{\lambda(1-\beta(1-\lambda))}{\alpha\kappa}, \\ \det(J_I) + \operatorname{tr}(J_I) < -1 \Longleftrightarrow \phi_\pi < -\frac{1}{\alpha\kappa}\left(1 + (1-\lambda)(1 - \beta\lambda + 2\beta + \alpha\kappa)\right) < 1. \end{cases}$$

Case 1 is satisfied under $\phi_\pi > 1$, and hence the standard steady-state REE (12) is determinate.

## E.2 Unintended steady-state REE

Define $J_U \equiv ((1-\lambda) B_U)^{-1}$. The unintended steady-state REE (14) is determinate if and only if matrix $J_U$ has both eigenvalues outside the unit circle, that is, either

$$\text{Case 1: } \begin{cases} \det(J_U) > 1 \Longleftrightarrow \frac{1}{\beta(1-\lambda)^2} > 1, \\ \det(J_U) - \operatorname{tr}(J_U) > -1 \Longleftrightarrow \frac{1}{\beta(1-\lambda)^2} - \frac{\beta+\alpha\kappa+1}{\beta(1-\lambda)} > -1 \Longleftrightarrow \frac{\lambda-(1-\lambda)(\alpha\kappa+\beta\lambda)}{\beta(1-\lambda)^2} > 0, \\ \det(J_U) + \operatorname{tr}(J_U) > -1 \Longleftrightarrow \frac{1}{\beta(1-\lambda)^2} + \frac{\beta+\alpha\kappa+1}{\beta(1-\lambda)} > -1, \end{cases}$$

or

$$\text{Case 2: } \begin{cases} \det(J_U) - \operatorname{tr}(J_U) < -1 \Longleftrightarrow \frac{1}{\beta(1-\lambda)^2} - \frac{\beta+\alpha\kappa+1}{\beta(1-\lambda)} < -1, \\ \det(J_U) + \operatorname{tr}(J_U) < -1 \Longleftrightarrow \frac{1}{\beta(1-\lambda)^2} + \frac{\beta+\alpha\kappa+1}{\beta(1-\lambda)} < -1. \end{cases}$$

Case 1 holds if and only if $\lambda - (1-\lambda)(\alpha\kappa + \beta\lambda) > 0$ (i.e., $\lambda > \bar{\lambda}$), and Case 2 does not hold for any $\lambda$. Hence, the unintended steady-state REE is indeterminate if and only if condition (17) is satisfied, while determinate if and only if condition (18) is satisfied.

# F Stability conditions

Stability conditions of the two steady-state REEs are obtained as follows.

## F.1 Standard steady-state REE

The Jacobian is

$$\begin{aligned} & D\left(T\left(a_2\right) - \left(a_2\right)\right) \\ = \; & \frac{1}{\alpha\kappa\phi_\pi + 1} \begin{bmatrix} -\lambda - \alpha\kappa\phi_\pi & \alpha(1-\lambda)(1-\beta\phi_\pi) \\ \kappa(1-\lambda) & (1-\lambda)(\beta+\alpha\kappa) - (\alpha\kappa\phi_\pi + 1) \end{bmatrix}. \end{aligned}$$

The Routh-Hurwitz stability criterion suggests that the ODE is stable if and only if the Jacobian has a characteristic polynomial,

$$\det(\gamma I - A) = \gamma^2 + \alpha_1\gamma + \alpha_0,$$

in which $\gamma$ denotes either eigenvalue of the Jacobian and the coefficients satisfy

$$\alpha_0 > 0, \quad \alpha_1 > 0.$$

These provide

$$\begin{aligned}
\phi_\pi &> 1 - \frac{\lambda\left(1-\beta\left(1-\lambda\right)+\alpha\kappa\right)}{\alpha\kappa}, \\
\phi_\pi &> 1 - \frac{\lambda\left(1-\beta\left(1-\lambda\right)+\alpha\kappa\right)}{\alpha\kappa} - \frac{\left(1-\lambda\right)\left(1-\beta+\alpha\kappa+2\beta\lambda\right)}{2\alpha\kappa}.
\end{aligned}$$

The second condition is redundant, and the first condition is the unique necessary and sufficient condition for the stability of the ODE.

## F.2 Unintended steady-state REE

The Jacobian is

$$D\left(T\left(a_2\right)-a_2\right) = \left(1-\lambda\right)B_U - I_2.$$

The Routh-Hurwitz stability criterion suggests that the ODE is stable if and only if the Jacobian has a characteristic polynomial,

$$\det\left(\gamma I - A\right) = \gamma^2 + \alpha_1\gamma + \alpha_0,$$

in which $\gamma$ is either of the eigenvalue of the Jacobian and the coefficients satisfy

$$\alpha_0 > 0, \quad \alpha_1 > 0.$$

That is,

$$\begin{aligned}
\lambda - \left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right) &> 0, \\
\lambda - \frac{\beta+\alpha\kappa-1}{\beta+\alpha\kappa+1} &> 0.
\end{aligned}$$

These conditions are simplified as follows:

$$\lambda > \frac{1}{2\beta}\left(\sqrt{\left(1-\beta+\alpha\kappa\right)^2+4\alpha\kappa\beta} - \left(1-\beta+\alpha\kappa\right)\right) > 0,$$

which is the unique necessary and sufficient condition of the stability of the ODE.

# G Proof of Proposition 11

Differentiating $\bar{r}^*$ in Eq. (16) with respect to $\pi^I$ yields

$$\begin{aligned}\frac{d\bar{r}^*}{d\pi^I} &= -1 + \frac{\lambda\phi_\pi\left(\alpha\kappa + 1 - \beta(1-\lambda)\right)}{(\phi_\pi - 1)\alpha\kappa + \lambda\left(\alpha\kappa + 1 - \beta(1-\lambda)\right)}(1-\eta) \\ &= -\frac{\lambda\phi_\pi\left(\alpha\kappa + 1 - \beta(1-\lambda)\right)}{(\phi_\pi - 1)\alpha\kappa + \lambda\left(\alpha\kappa + 1 - \beta(1-\lambda)\right)}(\eta - \bar{\eta}).\end{aligned}$$

Because $\alpha > 0$, $\kappa > 0$, $\phi_\pi > 1$, $0 < \beta < 1$, and $\lambda \in [0, 1]$, the coefficient of $(\eta - \bar{\eta})$ is strictly positive. This proves Proposition 11.

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